Composition of Labor Supply and Business Dynamism[*]

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Abstract

Population aging and the increase in the relative supply of college-educated workers have transformed the labor force in developed economies. How do these secular trends affect the characteristics of firms in the economy? To answer this question, I develop a general equilibrium model in which both workers and firms are heterogeneous. In the model, firms of different sizes rely on different types of workers due to capital-skill complementarity in production. I estimate the model using administrative linked employer-employee data from Germany. The model predicts that the changes in the labor force composition entail the reallocation of production towards firms with a larger capital stock, which tend to be older and less dynamic. The quantitative results indicate that the demographic trends can account for most of the recently documented shift in the size distribution of firms, the falling number of new firms, and the increasing market concentration. The patterns of business dynamism across German industries provide reduced-form empirical support for the model's predictions.

Keywords: Firm Dynamics, Startup Rate, Demographics, Concentration, Capital-Skill Complementarity
JEL classification: E13,E20,J11,L16,L26

1 Introduction

In recent decades, the structure of the labor force in developed economies has been fundamentally reshaped by two secular trends: population aging and the increase in the relative supply of college-educated workers. A large body of work documents the far-reaching impact of these long-run tendencies on various aspects of the economy, including income inequality, technological progress, monetary policy transmission, among many others. However, very little is known about the consequences of these demographic trends for the production side of the economy. How do these changes in the characteristics of workers affect the characteristics of firms in the economy?

In this chapter, I argue that the population aging and the increasing college attainment can lead to the concentration of production in larger, older, and less dynamic businesses. I develop and empirically validate a theory in which the composition of the labor force interacts with the life-cycle dynamics of firms. In the model, I incorporate both worker and firm heterogeneity, allowing firms of different sizes to employ different types of workers.[1] Through this channel, the changes in the composition of labor supply have heterogeneous effects on individual firms. The theory rests on complementarities in production between the physical capital of a firm and the human capital of its employees that are well-documented at the aggregate level (Krusell et al. 2000; Jaimovich et al. 2013). I propose a method to estimate these complementarities at the plant level using linked employer-employee data.

The theory helps to understand causes underlying recently documented secular decline in various measures of business dynamism in many developed economies. For example, the number of business startups has dropped, job creation and destruction rates have decreased, while economic activity has become more concentrated in large firms.[2] The model suggests that all these macroeconomic tendencies can be accounted by changes in the demographic structure of the workforce.

How can the secular trends in the composition of the labor force lead to a decline in business dynamism? As the population gets older and college education becomes more prevalent, there are more experienced and educated individuals in the labor market. Their labor becomes relatively less expensive, leading firms to increase the share of experienced and educated workers in the workforce.[3] The data reveal that experience and education are *complementary* to capital. At the plant level, the change in the composition of the workforce makes capital more productive, hence firms decide to accumulate more capital and increase em-

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[1]As is standard in the literature on business dynamism, I use the term *firm size* to refer to the number of currently employed workers. I use the terms firm, plant, and production unit interchangeably. The empirical evidence presented in the paper is based on establishment-level data. In the developed model, I consider single establishment firms.

[2]Davis et al. (2006) document a secular decline in measures of job creation and destruction in the U.S. economy. Haltiwanger et al. (2011) provide evidence of a secular decline in the rate of firm creation. Decker et al. (2016) document a decline in the number of high-growth firms. See Decker et al. (2014) and Akcigit and Ates (2019) for an overview. Calvino et al. (2015) document trends in firm creation across OECD countries. Bajgar et al. (2019) show that industries in the U.S. and in Europe are becoming more concentrated.

[3]The results are driven by a fall in wages relative to the marginal productivity of labor. The mechanism remains the same even if, due to technological change, the wage level increases, as long as its growth is slower than the growth of the marginal productivity.

ployment. However, capital accumulation takes time. Incumbent firms tend to be larger and have much higher capital stock than entrants[4]. As a result, incumbent firms tend to benefit much more from the increasing supply of the experienced and educated labor. As existing firms become larger and accumulate more capital, there is less space left for the startup businesses and the entry rate drops. Young firms tend to employ few workers at the beginning, but then grow quickly, creating most of the new jobs in the economy. Therefore, the falling number of new firms increases the average firm size, reduces job creation and leads to higher employment concentration in large firms. The new equilibrium features smaller number of larger, older, and less dynamic businesses.

The intuition discussed above is based on a general equilibrium model in which *both* firms and workers are heterogeneous. The model of heterogenous production units facing capital adjustment costs builds on Hopenhayn and Rogerson (1993), Khan and Thomas (2008), and Clementi and Palazzo (2016). However, in contrast to these papers, my model also includes worker heterogeneity. I assume that households supply three types of labor: raw labor, experienced labor, and educated labor.[5] Firms endogenously enter the economy and gradually accumulate physical capital over their life cycle. In every period, they face persistent idiosyncratic productivity shocks and can decide to shut down when they are no longer profitable. Production requires capital and the three types of labor, while the production technology allows for capital-experience and capital-education complementarities.

I parametrize the model using linked employer-employee panel data from Germany. The dataset is based on administrative records of employees and covers all establishments existing in Germany between 1976 and 2017.[6] I follow the literature and use a worker's age as a proxy for experience (Katz and Murphy 1992; Jaimovich et al. 2013). I classify workers who are 45 or older as experienced. Following Krusell et al. (2000), I assume that workers with a college or advanced degree supply educated labor.

I estimate the parameters governing firm entry, exit, and life-cycle dynamics using the simulated method of moments. The production complementarities are estimated in the following way. Firstly, I estimate a non-parametric relationship between firm size and workforce composition, controlling for firm characteristics (industry, age, the cohort of birth). Secondly, I choose the parameters of the production complementarities so that this relationship is the same in the model as in the data. The model replicates the German economy in the period 1976 - 1985.

The model allows me to analyze how the balanced growth path equilibrium is affected by exogenous changes in the supply of raw, experienced, and skilled labor. I modify the model parameters to reproduce the trends in the German labor market between the 1980s

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[4]See for instance Haltiwanger et al. (2013) for an overview of the life-cycle patterns of firm growth. Panel (A) in Figure 1 summarizes these patterns among German establishments.

[5]In what follows, I use the terms "skills" and "skilled labor" to denote skills acquired by doing, or experienced labor and well as skills acquired thorough formal education. I associate a skilled worker with a either an experienced worker or an college-educated worker. I also use the terms "plant", "establishment" and "production unit" interchangeably. In the context of the model developed in Section 3, the terms "production units" and "firms" are equivalent and I use them interchangeably.

[6]I use the Establishment History Panel created by the Institute for Employment Research. The dataset is described in detail in Section 4.

and 2010s. In the main experiment, I simultaneously alter the following aspects of labor supply: (i) the growth rate of the labor force, (ii) the relative supply of experienced workers, and (iii) the relative supply of college-educated workers.

The results of the main experiment show that the changes in the structure of the labor force can fully explain the increase in the average firm size and account for two-thirds of the drop in the startup rate. Moreover, they are responsible for 85% of the increase in market concentration, measured as the share of plants larger than 100 employees. Almost the entire effect is driven by the increase in the relative supply of experienced and educated workers. To understand how the labor force composition shapes business dynamism, I change one aspect at a time and examine the adjustments in the economy.

First, I consider a decline in the growth rate of the labor force, keeping the demographic composition intact.[7] The direct effect is that labor becomes scarcer and wages increase. Higher labor costs discourage potential entrants. A drop in the number of entrants leads to a lower job creation rate and a higher average firm size. The result echoes Hopenhayn et al. (2018) and Karahan et al. (2018), who use a model with homogenous workers.

The current paper, however, highlights additional general equilibrium effects. The slowdown in the labor force growth rate alone, without the concurrent changes in the age structure, would *not* lead to an increase in the average size of production units. Due to the production complementarities, the rise in the share of old firms in the economy induces a higher demand for experience and education. Since the structure of the labor supply is fixed, the higher demand is accommodated by an increase in wages of the two types of skilled labor. In response, firms switch to unskilled labor, accumulate less capital, and reduce their size. Therefore, the average size of production units decreases in the aftermath of the slowdown in the labor force growth rate. In the presence of the production complementarities, the skill composition of the labor force puts a constraint on the size distribution of firms in the economy.

Another important implication of my model is that, even for a fixed total size of the labor force, the changes in the relative supply of experienced and educated workers reshape the production side of the economy. According to the model, the increase in the relative supply of experienced or educated workers entails an increase in the average firm size and a drop in the startup rate. A similar increase in the relative supply of unskilled labor would have *opposite* consequences.

The crux of the matter lies in the production complementarities that I estimate using the micro-level data. The estimated relationship reveals that large firms rely heavily on experienced and educated workers. When the supply of the two types of workers increases, the large, capital-rich firms benefit the most from the ensuing changes in relative wages. On the other hand, young firms do not have much capital; hence they rely on unskilled labor that becomes relatively more expensive. They anticipate that in the future they will accumulate capital, employ more educated and experienced workers, and start benefiting from the demographic change. However, only half of all firms survive the first five years.[8] Because of that,

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[7]Decoupling the age structure of the population from the labor force growth rate may seem counterintuitive. However, this thought experiment is useful to illustrate the mechanism.

[8]More precisely, 46% of establishments close within the first five years after birth. This statistic is based on

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potential entrants heavily discount the future benefits against today's high prices of unskilled labor. As a result, the number of young firms in the economy falls.

I provide empirical support for the model using linked employer-employee data from Germany. Firstly, I demonstrate that the trends in the average firm size, entry, and concentration of employment are consistent with the model predictions. Secondly, I show that conditional upon age, the production units have become larger. Finally, I aggregate the establishment-level data into 3-digit industries and analyze at the industry level the relationship between the supply of experienced and educated workers and business dynamism. The data reveal that the industries that use experience and skills more intensively tend to be more concentrated, have a lower share of young firms, and are characterized by a higher average firm size.

In a related work, Engbom (2018) investigates how the population aging affects business dynamism. He shows that older workers are matched to better jobs, hence less likely to switch employers or become entrepreneurs. Due to lower worker mobility, *all firms* face a higher effective cost of job creation. As a result, business dynamism declines. The focus of the current paper is on the mechanism behind the rise in the average firm size and the increasing employment concentration in large firms, rather than worker reallocation per se. As mentioned above, the model predicts that the increase in the share of experienced and educated workers effectively increases the labor costs for startups, while reducing them for older production units.

In addition to a novel mechanism, the contribution of the paper is to account for the increase in relative supply of college-educated labor in addition to population aging.[9] I argue that, due to the imperfect substitutability of experienced and educated labor in production, accounting for the trends along both dimensions of human capital is essential for understanding the broad range of changes observed in developed economies. The model demonstrates that the reallocation of production towards large, productive, low-labor-share firms, recently documented in the U.S. and other economies by Kehrig and Vincent (2018) and Autor et al. (2019), was facilitated by concurrent trends in the composition of labor supply.

My work is broadly related to the rich literature on determinants of the college-wage premium and the returns to experience (Tinbergen 1956; Katz and Murphy 1992; Card and Lemieux 2001; Autor et al. 2003; Jeong et al. 2015). A common assumption in this strand of the literature is that the production side of the economy can be characterized by a representative firm. Consequently, the returns to human capital are determined by the aggregate supply of different groups of workers, the aggregate stock of physical capital, and the productivity of various types of labor. I show that the changes in the demographics of *firms* play an important role in explaining trends in wage distribution and income inequality.

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the German plant-level data for the period 1976-1985. See section 2 for details on the data set. Similar regularity holds in the U.S., as documented in the Business Dynamics Statistics Database.

[9]Jiang and Sohail (2019) and Salgado (2019) use a model of occupational choice to argue that the falling firm creation rate in the U.S. can be attributed to the rising returns to skills in the labor market. The latter rises entrepreneurs' outside options and discourages firm entry.

Outline

Section 2 describes the data and documents trends in the composition of the labor and business dynamism in Germany between 1975 and 2017. Section 3 describes the model. Section 4 deals with the model parameterization, while Section 5 discusses how the parameterized model is used to quantify the macroeconomic impact of changes in the composition of the labor force. Section 6 contains further empirical support for model predictions. Section 7 concludes.

2 Worker and Plant Demographics in Germany

In this section, I present the data set and give an overview of the relevant aspects of the German labor market. I begin with describing the main source of data and discussing the most important measures and definitions. Secondly, I report the changes in the composition of labor supply and trends in business dynamism in Germany between 1976 and 2017. Finally, I present the result of the regression analysis carried out to capture the relationship between worker and plant demographics.

2.1 Establishment History Panel (BHP)

The main source of data is the Establishment History Panel (BHP). The panel was created by the Institute for Employment Research (IAB) and is based on administrative records on health, pension and unemployment insurance of private sector employees.[10] The individual-level data is then aggregated into establishments based on unique establishment identification numbers.[11] Accordingly, the data contains the establishment-level information about the demographic structure of employees, wages, occupation, and education. The panel is a 50% random sample of all German establishments with at least one employee subject to social security as of 30 June of a given year. The sample consists of between 640,000 and 1.5 million establishments per year and covers the period between 1975 and 2017. In the analysis, I restrict attention to the establishments in West Germany with at least one employee (full-time or part-time).[12]

2.2 Measurement and Definitions

I follow the standard approach in the literature concerning the estimation of capital-skill complementarity (see , for instance, Griliches 1969; Krusell et al. 2000) and define *skilled workers* as employees having a college degree.[13] As is common in the literature, I use age as a

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[10]Civil servants, self-employed, and students are not recorded in the data set.

[11]The establishment is defined as "a regionally and economically delimited unit in which employees work. An establishment may consist of one or more branch offices or workplaces belonging to one company." (Schmucker et al. 2018, p. 17)

[12]For more details on the data set and its construction see Schmucker et al. (2018).

[13]Some degrees that typically would be earned at universities in other countries are obtained through vocational training in Germany. In order to be consistent with the literature on capital-skill complementarity, I include the vocational training as a part of college education.

proxy for experience (see, for instance, Katz and Murphy 1992; Jaimovich et al. 2013). The *experienced workers* are defined as employees of age 45 or above.[14] This is a parsimonious way to capture learning-by-doing over the course of worker's life. Most of the life-cycle increase in earnings takes place before age 45, suggesting that most of experience is accumulated before that age. Bayer and Kuhn (2018) document Using the German data that virtually all of the life-cycle wage growth attributed to worker's characteristics occurs before age 45.[15]

I define *plant size* as the total number of employees (both part-time and full-time). In order to correct for the mean-reversion bias in the estimates, I use the definition of size proposed by Davis et al. (1996). That is, I calculate plant size in period t as a simple average of the employee count in periods t and $t-1$. Formally, size of plant i in year t is defined as

$$n_{i,t} = 0.5\left(H_{i,t} + H_{i,t-1}\right), \tag{1}$$

where $H_{i,t}$ is the total number of employees in plant i in year t.

I define *education share* in a given plant as the share of workers who hold a college degree. Formally, education share in plant i in year t is defined as

$$S_{i,t} = \frac{H_{i,t}^s + H_{i,t-1}^s}{H_{i,t} + H_{i,t-1}}, \tag{2}$$

where $H_{i,t}^s$ marks the number of college-educated employees in plant i in year t. The *experience share* $E_{i,t}$ is defined analogously, as the share of employees who are 45 years old or older.

2.3 Changes in Labor Supply and Labor Demand in Germany

Figure 1 summarizes the trends in the composition of labor supply (Panels B, D, F), as well as the size distribution of plants in Germany between 1976 and 2017 (Panels A, C, E). In Panel (A) I present the relationship between plant age (the horizontal axis) and plant size. Each line corresponds to the set of plants set up in the year 1976, 1986, 1996, or 2016. In all age categories, plants established more recently are on average larger than plants established in the 1970s and the 1980s. Panel (C) reveals that, when comparing the size distribution of plants in the 1980s and 2010s, there was a significant shift towards larger units. Panel (E) illustrates that the startup rate (the share of plants of age 0 in the entire population of plants) has declined from above 5% in the 1980s to less than 3% in the 2010s. Similar secular tendencies have been documented in the U.S. and led scholars to worry about the performance of the American economy (see Decker et al. 2014; Akcigit and Ates 2019 for overview).

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[14]For the purpose of estimating the capital-experience complementarity, Jaimovich et al. (2013) define workers of age 30 or older as experienced. My results hold qualitatively for experience cutoff values of 30 years and 40 years old. I also performed robustness checks using potential experience defined as a difference between the current age of a worker and the approximate age of graduation. The qualitative results hold using this alternative measure of experience.

[15]This is also consistent with the literature on life-cycle earnings profiles based on the U.S data. For instance, Guvenen et al. (2017) document median wage by age for cohorts born between 1957 and 1983. Averaging their data on mean log income across all cohorts reveals that 98% of all lifetime increase occurs up to age 45.

Figure 1: Firm and worker demographics in Germany between 1976 and 2017

(A) Plant size over life cycle by cohort of birth
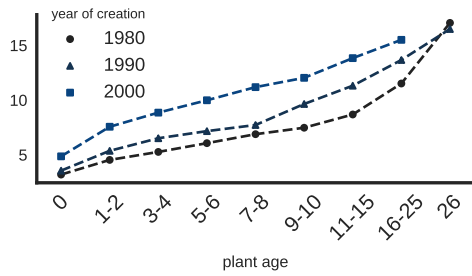
(B) Share of workers age 45 and older
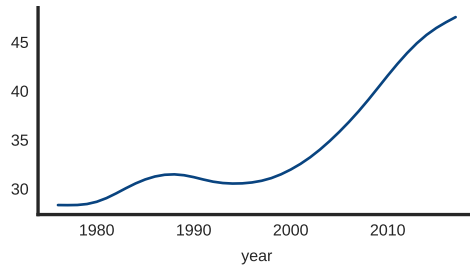
(C) Size distribution of plants
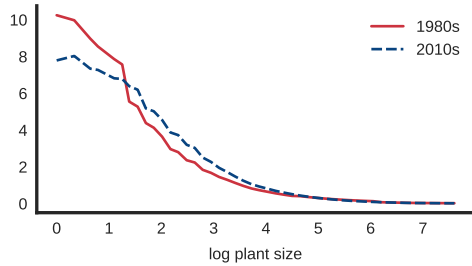
(D) Share of college-educated workers
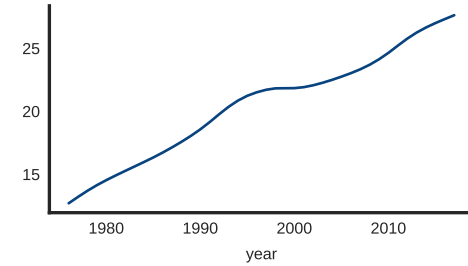
(E) Startup rate
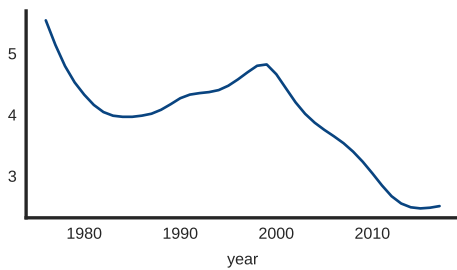
(F) Labor force growth rate
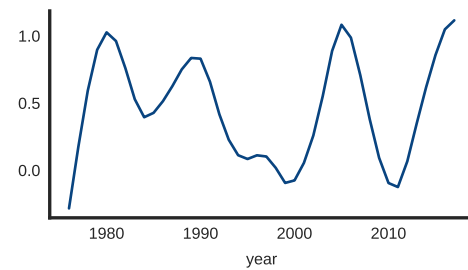
Notes: Calculations are based on the Establishment History Panel created by the IAB institute. Plant size is defined as the total number of employees. The share of college-educated workers is calculated as the total number of employees with a university degree or an advanced vocational training divided by the total number of employees. The startup rate is defined as the share of plants of age 0 in the total number of plants. All time series are smoothed with the HP filter with a smoothing parameter of 6.25.

The right-hand side of Figure 1 presents the trends in the supply side of the labor market. The demographic structure of the German labor force shifted towards older and more educated individuals. As depicted in Panels (B) and (D), the share of workers of age 45 or older has increased from 30% in 1980 to more than 45% in 2017, and the share of college-educated workers has increased from 15% to 26% during this period. At the same time, there is no visible trend in the growth rate of the labor force (Panel F).[16]

2.4 Relationship Between Worker and Plant Demographics

In this section, I document the relationship between the demographic structure of employees and plant size. To this end, I estimate the establishment-level regressions in which the share of experienced and college-educated employees is projected on a set of dummies indicating establishment's size. In order to control for the establishment characteristics, I include the following fixed effects: cohort of birth, industry, age, and year. I estimate the following regressions

$$E_{i,t} = \sum_g \beta_g^e \mathbf{1}\left(\text{size}_{i,t} \in g\right) + \{\text{year}_t, \text{industry}_i, \text{age}_i\} \text{ FE}, \tag{3}$$

$$S_{i,t} = \sum_g \beta_g^s \mathbf{1}\left(\text{size}_{i,t} \in g\right) + \{\text{year}_t, \text{industry}_i, \text{age}_i\} \text{ FE}, \tag{4}$$

where the depend variables $E_{i,t}$ and $S_{i,t}$ represent the shares of experienced and college-educated workers in plant i in year t. The indicator variable $\mathbf{1}\left(\text{size}_{i,t} \in g\right)$ is equal to one if the size of establishment i in year t falls into bin g.
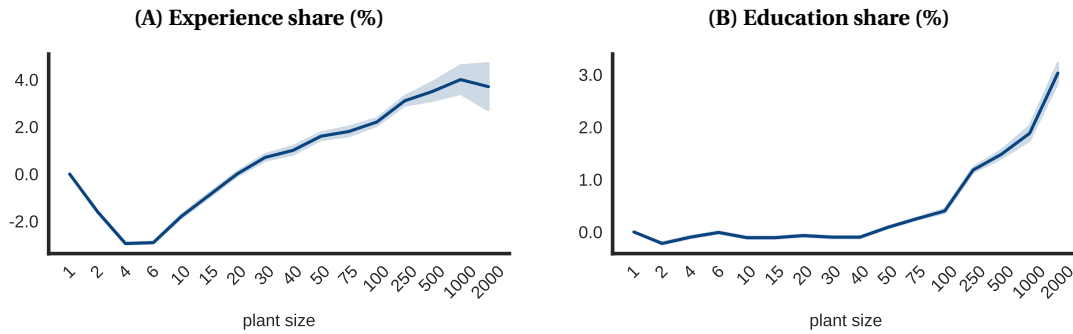
Figure 2 presents the estimated coefficients β_g of the dummy variables indicating size bins. The relationship between plant size and employee experience is non-monotonic: it is decreasing on the interval 1 to 4 and increasing for larger establishments. As presented in Panel (B), the share of educated workers is a convex function of plant size, increasing sharply at the upper tail of the size distribution.[17]

I incorporate the observed regularities in a reduced-form in my model. The documented patterns may stem from the capital-experience complementarity (see Jaimovich et al. 2013) or the capital-skill complementarity (as in Krusell et al. 2000). More recently, Blanas et al. (2019) analyse 30 developed countries and show that software and robots raised the demand for high-skilled and older workers, suggesting that college education and experience are complementary to this type of capital. Furthermore, it may be that in larger and more sophisticated organizations, more difficult problems arise in the production process. Consequently, these organization require more experienced and educated employees whose task is to solve these problems and manage the organization (see Garicano and Rossi-Hansberg 2006; Caicedo et al. 2019).

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[16]The aging of the German labor force is predominantly a result of the increasing participation rate and longer working life. Moreover, there is significant migration affecting the labor force growth rate.

[17]These results are in line with the empirical literature studying relationship between workers' human capital and firm's characteristics. For example, Haltiwanger et al. (2007) find positive association between firm productivity and worker skill profile.

Figure 2: The relationship between the plant size and employee demographics in the data.

(A) Experience share (%)

(B) Education share (%)

Notes: The blue solid lines present the estimated coefficients β_g from regressions (3) and (4) using the establishment panel data. In the regressions the share of the experienced workers (left panel) or the share of the college-educated workers (right panel) is projected on the set of dummies indicating plant size. The coefficients are normalized such that $\beta_1 = 0$. The shaded area represents 95% confidence intervals.

One may worry that the observed relationship between employee age (education) and employer size captures some unobserved worker characteristics that are unrelated to experience (schooling). In Appendix A.3, I study a subset of establishments for which a more detailed information on employee characteristics is available. I show that the relationship between plant size and employee experience (education) holds even after controlling for additional worker characteristics, including occupation and year of birth.

In conclusion, the changes in the composition of the labor force in Germany have been accompanied by declining business dynamism. Interestingly, there is a strong relationship between the characteristics of workers and the characteristics of the production units in the economy: larger units tend to employ much more experienced and much more educated workers. This suggests that the demographic trends may be one of the factors underlying observed changes in the production side of the economy. To explore this hypothesis, in the following sections I develop a general equilibrium model of firm dynamics; I estimate the model using the establishment-level data and use the model to quantify the impact of the demographic trends on business dynamism.

3 Model

This section describes the model of interactions between heterogeneous plants and heterogeneous workers. I specify the household side of the model to allow for a simple representation of the following secular trends in the composition of the labor force: a slowdown in the labor force growth rate, population aging, and an increasing supply of college-educated workers.

The production side of the economy builds on Clementi and Palazzo (2016). The model features production units indexed by productivity and the stock of capital. Production units

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endogenously enter the economy, gradually accumulate capital and can decide to shut down. They face persistent idiosyncratic productivity shocks. Production requires capital and different types of labor. The production function allows for the complementarities in production between the labor type and the plant type. All inputs for the production are traded in competitive markets and there is no aggregate uncertainty. I introduce the general equilibrium following Khan and Thomas (2008).

I begin with the description of households, then follow with the production side, aggregation, and equilibrium.

3.1 Households

Time is discrete. Next period's variables are denoted with primes. The economy is populated by a large family consisting of measure N of infinitely-lived, identical members. Household size grows over time at rate g_n so that $N' = (1 + g_n)N$. Household members derive utility from consumption and suffer disutility from supplying labor. Each household member is endowed with a stock of human capital. There are three aspects of human capital. Household members can supply raw labor l, experienced labor e, and educated labor s. The family head decides on the labor supply of each worker.

The instantaneous utility function of each household member is

$$U_l(c, n_l, n_e, n_s) = \log c - \frac{\psi_l}{1+\eta} n_l^{1+\eta} - \frac{\psi_e}{1+\eta} n_e^{1+\eta} - \frac{\psi_s}{1+\eta} n_s^{1+\eta},$$

where c denotes consumption and n_x marks the supplied hours of labor of type $x \in \{l, e, s\}$. $1/\eta$ is the Frish elasticity of labor supply. Parameters ψ_l, ψ_e, ψ_s govern the steady state supply of the three types of labor.

The family stores its wealth in one-period shares in plants. Measure $b(z, k)$ describes the number of shares in plants of type $(z, k) \in \mathcal{S}$ that the household owns, where z and k denote the plant-level productivity and capital stock. The production units are described in detail below. The household chooses the level of consumption per capita, the supply of the three types of labor, and the firm equity holdings, while taking as given the price $q_0(z, k)$ of the current shares (which includes dividends), the price $q_1(z, k)$ of the new shares, the wages w_l, w_e, w_s and the price of the final good p.

The household solves the following maximization problem

$$V^H(b) = \max_{c, n_l, n_s, n_s, b'} N \times U(c, n_l, n_s, n_s) + \beta V^H(b'), \tag{5}$$

subject to the budget constraint

$$pNc + \int_{\mathcal{S}} q_1(z, k) b'(\mathrm{d}[z \times k]) = N(n_l w_l + n_e w_e + n_s w_s) + \int_{\mathcal{S}} q_0(z, k) b(\mathrm{d}[z \times k]). \tag{6}$$

The optimal choice of labor supply equalizes the utility of an additional wage income with the disutility of an additional hour of work. The first-order conditions describing the optimal labor supply by the household members are given by

$$w_l = pc\psi_l n_l^\eta, \quad w_e = pc\psi_e n_e^\eta, \quad w_s = pc\psi_s n_s^\eta. \tag{7}$$

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The total supply of raw, experienced, and skilled labor services is $N \times n_l$, $N \times n_e$, $N \times n_s$, respectively.

Let λ denote the Lagrange multiplier on the budget constraint. The first-order condition for consumption is

$$\frac{1}{c} = \lambda p, \tag{8}$$

meaning that the marginal gain from an additional unit of income is equal to the marginal utility from consumption. The first-order condition for the equity holdings satisfies

$$\lambda q_1(z, k) = \beta \lambda' q_0'(z, k), \tag{9}$$

for all shares (z, k) and equates the marginal cost of foregoing consumption with the future returns on investment in equity. Conditions (8) and (9) give rise to the Euler equation

$$\frac{c'}{c} = \beta \frac{p}{p'} \frac{q_0'(z, k)}{q_1(z, k)}, \tag{10}$$

for all (z, k). The Euler equation states that at the optimum, the household is indifferent between allocating resources to consumption in the current period or to consumption in the next period (through savings in equity).

3.2 Plants

This section describes the production side of the economy. At the beginning of each period, there is an endogenous mass of incumbent plants. Incumbents can decide whether to continue operating or to exit the market. Continuing plants choose investment subject to capital adjustment costs. Additionally, in each period there is an endogenous mass of entrants.

The plants are characterized by the idiosyncratic productivity $z \in [\underline{z}, \bar{z}]$ and by the beginning-of-period capital stock $k \in [\underline{k}, \bar{k}]$. Plant-specific productivity evolves according to the following AR(1) process

$$\log(z') = \bar{\mu}_z + \rho_z \log(z) + \sigma_z \varepsilon, \qquad \varepsilon \sim \mathcal{N}(0, 1), \tag{11}$$

where $\bar{\mu}_z$ denotes the mean level of productivity, ρ_z is the persistence of the process, and σ_z is the standard deviation of the productivity shocks. In what follows, $F_z(z'|z)$ denotes the conditional distribution of next period's productivity z', conditional on current period's productivity z.

Production requires capital and the three types of labor. Let $L(l, e, s; z, k)$ denote a composite of labor services supplied by all workers employed in a plant of type (z, k) in period t. As described in detail below, the functional form of the labor composite depends on the plant characteristics. This assumption captures the production complementarities between employees' human capital and plant's type. Each plant has access to the following production function

$$y(z, k, l, e, s) = z k^\alpha L_t(l, e, s; z, k)^\nu,$$

where $\alpha, \nu \in (0,1)$ govern the elasticities of output with respect to capital and labor, respectively, and L is the labor composite. The latter is given by

$$L(l,s,e;z,k) = \left[l^{\frac{\theta-1}{\theta}} + \bar{A}_e \times A_e(z,k) e^{\frac{\theta-1}{\theta}} + \bar{A}_s \times A_s(z,k) s^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}}, \tag{12}$$

where $\bar{A}_e \times A_e(z,k)$ and $\bar{A}_s \times A_s(z,k)$ capture the marginal productivity of experienced and skilled labor employed at a firm of type (z,k). The productivity schedules consist of two ingredients. Firstly, $\bar{A}_e$ and $\bar{A}_s$ capture the time-varying factor-specific aggregate efficacy of the two types of labor. The second ingredient captures the capital-skill complementarities, $A_e(z,k)$ and $A_s(z,k)$, and reflect the comparative advantage of experienced and educated workers, respectively, when employed at a firm of type (z,k). Productivity of raw labor is normalized to unity in all periods. Parameter θ captures the elasticity of substitution between different types of labor. I restrict attention to the case $\theta \geq 1$ in which the labor inputs are imperfect substitutes.[18]

I capture the capital-skill complementarities by allowing the productivity schedules $A_e(z,k)$ and $A_s(z,k)$ depend on the firm type (z,k). The assumption states that the output generated by one unit of skilled or experienced labor depends on plant's capital stock and productivity. For example, the productivity of a computer scientist (skilled labor) depends on the quantity and quality of the IT equipment at her disposal, whereas the value added of a manager (experienced labor) depends on the scale of the organization and the complexity of problems she is assigned to solve.

This functional form of the production function allows me to capture in a tractable way the key intuition behind the capital-skill complementarities: the productivity of experienced and educated labor depends on the quantity and quality of the capital. The above specification of the production function is very flexible and puts little a priori restrictions on the shape of the production complementarities. One important restriction is that the shape of complementarities, the functions $A_e(z,k)$ and $A_s(z,k)$, is time-invariant. This will be addressed in a grater detail in the quantitative experiments, Section 5.

The approach is also agnostic to the microeconomic mechanism underlying the complementarities. Explaining *why* skills acquired by learning-by-doing and formal education are complementary to capital is beyond the scope of this paper. I take the existence of these complementarities as given and I estimate them using the matched employer-employee data from Germany. In the estimation procedure, I choose the values of the productivity schedules $A_e(z,k)$ and $A_s(z,k)$, for all z, k, such that the model replicates the relationship between plant size and the demographic structure of its employees in the data. Thanks to the tractable specification, the production function can be estimated using only the information about the the skill composition of employees across establishments.

An alternative way of modeling the capital-skill complementarity is a nested CES structure in the production function. This allows the elasticity of substitution between capital

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[18]This is a standard assumption in the labor economics literature. The estimates of the elasticity of substitution between college- and high-school-educated labor vary from 1.4 in Katz and Murphy (1992), to 1.5 in Johnson (1997), to 2.5 in Card and Lemieux (2001). The latter work provides estimates of the elasticity of substitution between experienced and inexperienced workers in the range of 4 to 6.

and skilled labor to be different from the elasticity between capital and unskilled labor (see Griliches 1969; Krusell et al. 2000). In the current paper, the skills of workers are two dimensional – experience and education – making the nested CES structure less tractable. Moreover, the standard nested CES structure cannot easily replicate the salient features of the data that large firms tend to employ much more experienced and much more educated workers (see Figure 3). The reason is that, in the heterogeneous firm dynamics model developed in the current paper, there are two reasons for a firm to be large: a high capital stock or a high productivity. However, in a standard nested CES structure, for a given level of capital, the more productive the firm, the *lower* is the share of skilled workers it employs.[19] Finally, the specification of the production function assumed in the current paper makes it possible to estimate the production function using only information on the number and the demographic composition of employees. I do not need to measure capital at the firm level which has proven to be a notoriously difficult task. See Section 4 for more details on the estimation procedure.

Static problem of incumbent plants.

At the beginning of each period, all incumbent plants produce the final good. To this end, they solve the following static maximization problem

$$\pi = \max_{L} \left\{ pzk^{\alpha}L^{\nu} - W(z,k)L \right\}, \tag{13}$$

where $\pi(z,k)$ is the current profit, and $W(z,k)$ is the minimal cost of employing one unit of the composite labor L. The optimal choice of the composite labor satisfies

$$L^* = \left(\frac{pzk^{\alpha}\nu}{W(z,k)} \right)^{\frac{1}{1-\nu}}.$$

The above condition states that the plant hires additional workers up to the point in which the marginal gain of an additional unit of labor equals its marginal cost. The gain is proportional to plant's effective productivity zk^{α}. The marginal cost $W(z,k)$ depends on the firm type (z,k), since the production complementarities imply that establishment's characteristics (z,k) determine the skill composition of employees.

Given the optimal choice of the labor composite L^*, the plant decides how much services of the three labor types to hire to minimize the total labor cost. The cost-minimizing allocation satisfies[20]

$$e = \left(\bar{A}_e A_e(z,k) \right)^{\theta} \omega_e^{-\theta} l, \qquad s = \left(\bar{A}_s A_s(z,k) \right)^{\theta} \omega_s^{-\theta} l, \tag{14}$$

where

$$l = L^* \times \left[1 + \left(\bar{A}_e A_e(z,k) \right)^{\theta} \omega_e^{1-\theta} + \left(\bar{A}_s A_s(z,k) \right)^{\theta} \omega_s^{1-\theta} \right]^{-\frac{\theta}{\theta-1}} \tag{15}$$

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[19]Similar point is made by Jaimovich et al. (2013) in the context of the aggregate production function and heterogeneity in the cyclical volatility of employment. To understand this intuition, consider a limiting case of Leontiev production function. In this case, the efficiency units of skilled labor used in the production are always exactly equal to the efficiency units of capital. Consequently, for a given capital stock, all increase in size stemming from productivity improvements will be accommodated by unskilled labor.

[20]The derivations can be found in Appendix B.1.

and $\omega_e = \frac{w_e}{w_l}$, $\omega_s = \frac{w_s}{w_l}$ denote the experience and skill wage premium, respectively. The above conditions define the most cost-effective way of splitting the total labor input L^* into raw, experienced, and skilled labor. Note that the allocation of the labor demand between the three types of labor depends on the plant type (z, k). For a plant of type (z, k), the minimal cost of hiring one unit of the composite labor is

$$W(z, k) = w_l \left[1 + \left(\bar{A}_e A_e(z, k) \right)^\theta \omega_e^{1-\theta} + \left(\bar{A}_s A_s(z, k) \right)^\theta \omega_s^{1-\theta} \right]^{-\frac{1}{\theta-1}}. \tag{16}$$

The unit cost of composite labor is a weighted average of the wages of the three labor types. The weights depend on productivity $\left(\bar{A}_e A_e(z, k) \right)$ and $\left(\bar{A}_s A_s(z, k) \right)$ which, in turn, depend on plant's type. The type of labor that is the most productive receives the highest weight in the total wage cost. The more a plant relies on one type of labor, the more sensitive it is to changes in the corresponding wage.

Continuation and investment decisions.

In each period, after producing the final good, incumbent plants incur a stochastic, i.i.d. overhead cost $c_f \sim G_f$ expressed in terms of output. After observing the realization of the shock, incumbents decide whether to shut down or to pay the cost and continue operating. Upon exit, the plant sells the remaining stock of capital $(1 - \delta)k$ net of the destruction costs $g(-(1 - \delta)k, k)$. The exit value is given by

$$V^x(k) = p \left[(1 - \delta)k - g(-(1 - \delta)k, k) \right].$$

For large realizations of the cost c_f, the continuation value of the plant may fall below the value of selling its capital stock. In this case, the plant will decide to exit.

Plant that decided to continue operating invests i units of capital. The capital stock evolves according to

$$k' = (1 - \delta)k + i. \tag{17}$$

The plant with capital k undertaking investment i pays the adjustment costs of $g(i, k)$ units of output.

At the beginning of period, the value of the incumbent plant $V(z, k)$ equals the sum of the current profit $\pi(z, k)$ and plant's continuation value that depends on the decision whether to continue operating or to exit

$$V(z, k) = \pi(z, k) + \int_{\mathbb{R}} \max \left\{ V^x(k), \tilde{V}(z, k) - p c_f \right\} G_f(\mathrm{d}c_f). \tag{18}$$

$\tilde{V}(z, k)$ denotes the value of the plant that decided to continue operating. The integral stems from the stochastic nature of the operating costs c_f. The plant exits whenever the continuation value net of the operating costs $\tilde{V}(z, k) - c_f$ falls below the value of exit $V^x(k)$. The value of the continuing plant is given by

$$\tilde{V}(z, k) = \chi V^x(z, k) + (1 - \chi) \max_i \left[-pg(i, k) + \frac{1}{1+r} \int_Z V(z', k', \mu) F_z(\mathrm{d}z'|z) \right], \tag{19}$$

15

where χ denotes an exogenous destruction probability (time-invariant and common across plant). Plant's discount factor $\frac{1}{1+r}$ is determined in the equilibrium.

Let $\bar{c}_f(z,k)$ be the threshold value of the cost at which the plant decides to exit. The threshold is given implicitly by

$$\bar{c}_f(z,k) = \frac{\tilde{V}(z,k) - V^x(k)}{p}. \tag{20}$$

The exit probability X equals the probability that the cost realization exceeds the above threshold

$$X(z,k) \equiv 1 - G_f\left(\bar{c}_f(z,k)\right). \tag{21}$$

In expectations, the cost paid by the plant satisfies

$$\tilde{c}_e(z,k) = \int_0^{\bar{c}_f(z,k)} c_f \, G_f(\mathrm{d}c_f). \tag{22}$$

Entry.

Let me now describe how firms are created in the economy. A salient feature of the firm-level data in all developed economies is that the start-up business are on average much smaller than incumbents. However, conditional on survival, young firms grow rapidly creating most of the new jobs in the economy (Haltiwanger et al. 2013; Decker et al. 2014). I specify the entry problem in the economy to capture these features of the data.

One explanation for the small size and the subsequent rapid growth of young firms is a financial friction (Albuquerque and Hopenhayn 2004; Clementi and Hopenhayn 2006; Buera 2009; Buera and Shin 2011). Due to market imperfections, prospective entrepreneurs face a constraint on the amount of capital they can borrow. As a result, young firms tend to operate at a scale that is below the optimal one. However, as firms become larger and more established, they slowly overcome market imperfections and gain easier access to capital.

With this intuition in mind, I specify the entry problem as follows. I assume that in each period, there is a mass M^e of potential entrants. Each prospective entrepreneur decides whether to pay a fixed cost of $c_e \geq 0$ units of the final good and enter the market. After paying the fixed cost and choosing the value of capital, the entrant receives a draw of initial productivity $z_e \in Z_e$ from the cdf F_e. Next, each potential entrant chooses the level of initial capital, subject to adjustment costs $g_e(k)$, an increasing and convex function. From then on, the entrant behaves like one of the incumbent production units described earlier.

Therefore, entrants differ with respect to their initial productivity. Although more productive entrants tend to choose a higher capital stock, the adjustment costs g_e induce all startups to choose capital that is lower than the unconstrained optimum. Intuitively, one can think of this environment as a reduced-form penalty function approach to approximating an imperfect capital market in which contracts are not fully enforceable and prospective entrepreneurs face collateral constraints (Marcet and Marimon 1992; Cagetti and De Nardi 2006).

Let V_e denote the value of entry defined as

$$V_e(z_e) = \frac{1}{1+r} \max_k \left[-pk - pg_e(k) + \int_Z V(z,k) F_z(\mathrm{d}z|z_e) \right].$$

(23)

The mass of entrants is determined endogenously by the free entry condition. The potential entrant with initial productivity z_e decides to enter if and only if $V_e(z_e) \geq pc_e$. In what follows, $k^*(z_e)$ denotes the optimal capital choice of entrant receiving and initial productivity draw z_e.

3.3 Aggregation

The aggregate state of the economy consists of the plant measure μ describing the distribution of plants over the idiosyncratic state: the current productivity $z \in [\underline{z}, \bar{z}]$ and the beginning-of-period capital stock $k \in [\underline{k}, \bar{k}]$. The measure μ is defined on the Borel algebra $\mathscr{S}$ for the product space $[\underline{z}, \bar{z}] \times [\underline{k}, \bar{k}]$.

The measure μ includes surviving incumbents as well as startups, and evolves according to the following law of motion: for any measurable set $A \subset \mathscr{S}$ such that $z' \in A$

$$\mu'(A) = (1-\chi) \underbrace{\int_{(z,k):\, (k^*(z,k)) \in A} (1 - X(z,k)) \, F_z(z'|z) \, \mu(\mathrm{d}[z \times k])}_{\text{incumbents that choose } k^* \in A \text{ and transition to } z' \in A}$$

$$+ M \underbrace{\int_{z':\, z' \in A} \int_{z_e:\, k^*(z_e) \in A, V_e(z_e) \geq pc_e} G_e(\mathrm{d}z_e) F_z(z'|z_e)}_{\text{entrants that draw } z' \in A, \text{ choose } k^* \in A}.$$

(24)

The first line captures incumbents in the current state (z,k) that decided to continue, chose capital k^* and transitioned from z to z', for all pairs $(z',k^*) \in A$. The second line adds the mass of entrants that drew initial productivity z_e, choose capital k^* and draw next period productivity z' such that $(z',k^*) \in A$.

The aggregate variables are defined as follows. The real aggregate output is given by the production net of the operating and adjustment costs

$$Y = \int_{\mathscr{S}} z k^\alpha L^*(z,k)^\nu \mu(\mathrm{d}[z \times k])$$

$$- (1-\chi) \int_{\mathscr{S}} (1 - X(z,k)) \left[\tilde{c}_f(z,k) + g(i(z,k),k) \right] \mu(\mathrm{d}[z \times k])$$

$$- \chi \int_{\mathscr{S}} (1 - X(z,k)) \left[g(-(1-\delta)k, k) \right] \mu(\mathrm{d}[z \times k])$$

$$- \int_{\mathscr{S}} X(z,k) \left[g(-(1-\delta)k, k) \right] \mu(\mathrm{d}[z \times k])$$

$$- Mc_e - M \int_{\underline{z}}^{\bar{z}} g_e(k(z_e)) G_e(\mathrm{d}z_e).$$

The first line captures the output of a firm of type (z,k) and then integrates over all possible types. The integration is with respect to equilibrium measure of firms μ that dictates the

"number" of firms of each type. The following lines subtract capital adjustment costs for continuers and exitors, and subtracts the entry costs and adjustment costs paid by startups.

The aggregate net investment equals the sum of investments of incumbents and entrants, net of capital sold by exitors

$$I = (1-\chi)\int_{\mathscr{S}}(1-X(z,k))\left(k^*(z,k)-(1-\delta)k\right)\mu\left(\mathrm{d}[z\times k]\right) + M\int_{\underline{z}}^{\bar{z}}k(z_e)G_e(\mathrm{d}z_e)$$

$$-\int_{\mathscr{S}}\left[\chi(1-X(z,k))(1-\delta)k + X(z,k)(1-\delta)k\right]\mu\left(\mathrm{d}[z\times k]\right). \tag{25}$$

The aggregate resource constraint in the economy is

$$Nc = Y - I.$$

3.4 Recursive Equilibrium

A recursive competitive equilibrium is a set of functions V, $\tilde{V}$, $\pi(z,k)$, k^*, L^*, X, l, e, s, c, b, n_l, n_e, n_s, firm measure μ and prices p, w_l, w_e, w_s such that, given prices,

1. V, $\tilde{V}$ and $\pi(z,k)$ solve the plant's optimization problems (13), (18), (19), and X, k^*, L^*, l, e, s are the associated policy functions.

2. V^H solves the household's optimization problem (5) and c, b, n_l, n_e, n_s are the associated policy functions.

3. Labor markets clear

$$N(1-v_e-v_s)\,n_l = \int_{\mathscr{S}}l(z,k)\mu\left(\mathrm{d}[z\times k]\right),$$

$$Nv_e n_e = \int_{\mathscr{S}}e(z,k)\mu\left(\mathrm{d}[z\times k]\right),$$

$$Nv_s n_s = \int_{\mathscr{S}}s(z,k)\mu\left(\mathrm{d}[z\times k]\right).$$

4. Equity market clears

$$b'(z',k') = \mu'(z',k'), \qquad \text{for all } (z',k')\in\mathscr{S}.$$

5. Final good market clears by Walras law.

Balanced growth.

In what follows I restrict attention to balanced growth equilibria. An equilibrium is said to be balanced growth path if the prices p, w_l, w_e, w_s are time-invariant and the measure μ satisfies

$$\frac{\mu'(A)}{N'} = \frac{\mu(A)}{N}, \qquad \text{for all measurable } A\in\mathscr{S},$$

meaning that the plant measure normalized by population is stationary. Let $\hat{\mu}$ denote the normalized firm measure. Given this normalization, all variables in the economy are stationary.

Following Hopenhayn (1992), I rewrite the law of motion for the normalized firm measure

$$\hat{\mu}' = \frac{(1-\chi)}{1+g_n} P\hat{\mu} + \hat{M}' v, \tag{26}$$

where P is a bounded linear operator such that, conditional on policy functions $X(z,k)$ and $k^*(z,k)$, for every measurable set $A \in S$

$$P(z,k,A;X,k^*) = \begin{cases} \int_{z' \in A} F(\mathrm{d}z'|z) & \text{if } X(z,k) = 0 \text{ and } k^*(z,k) \in A \\ 0 & \text{otherwise} \end{cases} \tag{27}$$

and the measure of entrants satisfies

$$v(A) = \int_{z \in A} \int_{k \in A} G_e(\mathrm{d}z) G_k(\mathrm{d}k). \tag{28}$$

Intuitively, one can think about plants in the economy as aggregate resources. The larger the "stock" of plants (i.e. the number of plants), the more output can be produced. The plant stock is accumulated according to the law of motion (26). The population growth rate g_n can be interpreted as a "depreciation" of the plant stock.

From the stationarity of $\hat{\mu}$, it follows that

$$\hat{\mu} = \hat{M} \left(I - \frac{1-\chi}{1+g_n} P \right)^{-1} v = \hat{M} \sum_{t=0}^{\infty} \left(\frac{1-\chi}{1+g_n} P \right)^t v, \tag{29}$$

where P^t is the t-fold composition of P with itself and P^0 is the identity operator.

The aggregate output per capita is defined as

$$\begin{aligned}
\hat{Y} = &\int_{\mathscr{S}} z k^{\alpha} L^*(z,k)^{\nu} \hat{\mu} \left(\mathrm{d}[z \times k] \right) \\
&- (1-\chi) \int_{\mathscr{S}} (1 - X(z,k)) \left[\tilde{c}_f(z,k) + g(i(z,k),k) \right] \hat{\mu} \left(\mathrm{d}[z \times k] \right) \\
&- \chi \int_{\mathscr{S}} (1 - X(z,k)) \left[g(-(1-\delta)k,k) \right] \hat{\mu} \left(\mathrm{d}[z \times k] \right) \\
&- \int_{\mathscr{S}} X(z,k) \left[g(-(1-\delta)k,k) \right] \hat{\mu} \left(\mathrm{d}[z \times k] \right) - \hat{M}^e c_e.
\end{aligned} \tag{30}$$

The remaining aggregates are defined analogously. In per capital terms, the clearing of the markets for the three types of labor requires

$$\begin{aligned}
n_l &= \int_{\mathscr{S}} l(z,k) \hat{\mu} \left(\mathrm{d}[z \times k] \right), \\
n_e &= \int_{\mathscr{S}} e(z,k) \hat{\mu} \left(\mathrm{d}[z \times k] \right), \\
n_s &= \int_{\mathscr{S}} s(z,k) \hat{\mu} \left(\mathrm{d}[z \times k] \right).
\end{aligned} \tag{31}$$

4 Estimation and Model Fit

In this section, I describe how I bring the model to the data. The goal of the paper is to investigate the macroeconomic impact of the changes in the demographic structure of the labor force in Germany between the 1980s and 2010s. To this end, I parameterize the model so as to replicate the German economy in the period 1976 - 1985. Firstly, I discuss the assumptions concerning the functional forms used in the quantitative model. Secondly, I describe the calibration strategy. Finally, I discuss the model fit.

4.1 Functional Forms

The distribution of the operating costs G_f is assumed to be log-normal with the mean $\bar{\mu}_f$ and the standard deviation σ_f. The capital adjustment cost has a fixed and a convex part. Following Cooper and Haltiwanger (2006), I assume the functional form

$$g(i,k) = \begin{cases} \zeta_0 k + \zeta_1 \frac{i^2}{k} & \text{if } \left|\frac{i}{k}\right| > 0.01, \\ \zeta_1 \frac{i^2}{k} & \text{otherwise.} \end{cases} \tag{32}$$

The first component of the adjustment costs does not depend on the value of investment. It aims to capture any disruption to the production process or other costs resulting from undertaking new investment, irrespective of its scale. It is paid only if the investment rate is larger than 1%. The second component is quadratic in the value of investment capturing the intuition that larger investment projects tend to be more than proportionally more difficult to complete. For entrants, the adjustment costs reads

$$g_e(k) = \zeta_e \exp(k) \tag{33}$$

The fixed component of the investment costs for startups is subsumed in the entry cost c_e. The initial productivity distribution for entrants, G_e, is assumed to be Pareto with the shape parameter ξ_e

$$F_e(z_e) = 1 - \left(\frac{z}{z_e}\right)^{\xi_e}.$$

I discretize the productivity grid using the Rouwenhorst method with 19 grid points. I use 501 points for the grid of capital spaced logarithmically over $[0.001, 1]$. I assume that the set of feasible initial productivity draws is $Z_e = [\mu_z - 3\sigma_z, \mu_z + 3\sigma_z]$ and is discretized using 501 equidistant points.

4.2 Calibration Strategy

A subset of parameters is calibrated externally, based on the literature. The remaining parameters are then estimated jointly with the simulated method of moments (SMM) using the linked employer-employee data from Germany. The parameters of the production complementarities between worker type and firm type are estimated through indirect inference. The procedure ensures that the relationship between the demographic structure of employees and the plant size in the model replicates the patterns estimated using the BHP establishment panel.

Table 1: Externally Calibrated Parameters

Parameter		Value	Target
I. Households			
β	discount factor	0.96	real interest of 4%
η	inverse of Frish elast.	0.25	standard RBC value (King and Rebelo 1999)
v_e	share of experienced workers	0.62	value for Germany in the 1980s
v_e	share of skilled workers	0.15	value for Germany in the 1980s
ψ_l	disutility, raw labor	1.13	normalization, 1 efficiency unit per worker
ψ_e	disutility, experienced labor	1.06	normalization, 1 efficiency unit per worker
ψ_s	disutility, skilled labor	1.78	normalization, 1 efficiency unit per worker
II. Incumbent plants			
α	output elasticity of capital	0.21	model-free estimates in Bachmann and Bayer (2014)
v	output elasticity of labor	0.56	model-free estimates in Bachmann and Bayer (2014)
δ	depreciation rate	0.09	model-free estimates in Bachmann and Bayer (2014)
χ	exogenous exit rate	0.01	exit rate of plants with 250+ employees
III. Entrants			
c_e	entry costs	101.54	normalization, unit price of final good

Notes: I define experience share as the share workers of age 45 or older. I define education share as the share of workers who hold a college degree. The calculations are based on the BHP panel. Self-employed, unemployed, and public sector employees are not included in the data set.

Externally calibrated parameters.

The externally calibrated parameters are summarized in Table 1. One period in the model corresponds to one year. I set the discount factor β to 0.96, implying an annual interest rate of 4%.

I choose the parameters governing shares of household types such that the relative supply of experience equals 0.30 and the relative supply of skills equals 0.15, matching the shares of experienced and college-educated workers in the German data. The parameters ψ_l, ψ_e, ψ_s governing the disutility of labor of different workers are set such that each worker supplies one efficiency unit of labor. Frish elasticity of labor supply is set to 4, implying $\eta = 0.25$. The elasticity of output with respect to capital is set to $\alpha = 0.2075$ and with respect to labor to $v = 0.5565$. These values are directly estimated using the German firm-level data by Bachmann and Bayer (2014). The depreciation of physical capital is set to $\delta = 0.09$, the value calculated directly using the German national accounting data by Bachmann and Bayer (2014). The cost of entry is normalized to ensure that the price of the final good equals one. The exogenous destruction probability $\chi = 0.01$ is set to the value of the exit rate among plants larger than 250 employees.

Production complementarities.

The interactions between the demographic structure of the labor force and business dynamism depend crucially on the parameters of the production complementarities. I estimate the productivity schedules $A_e(z, k)$ and $A_s(z, k)$ to ensure that the model replicates the relationship between plant size and employee composition in the BHP panel in the period 1976 - 1985.

The demand for experienced and educated labor services is given by equations (14) can be written as

$$\log e(z, k) - \log l(z, k) = \theta \log \bar{A}_e - \theta \log w_e + \theta \log A_e(z, k) \tag{34}$$

and

$$\log s(z, k) - \log l(z, k) = \theta \log \bar{A}_s - \theta \log w_s + \theta \log A_s(z, k) \tag{35}$$

The firm type (z, k) is unobservable in the data. However, in the model there is a one-to-one mapping between the firm type and the firm's total number of employees, l. The latter is directly measurable in the data.

Consider an establishments i in year t, operating in sector j. The establishments was created in year c. Let G denote a family of size bins, where a size bin $g \in G$ is an interval $[g_1, g_2) \subset \mathbb{R}^2$ such that $0 \leq g_1 \leq g_2 < \infty$. I estimate the following regression

$$\log e_{i,t} - \log l_{i,t} = \alpha + \sum_{g \in G} \beta_g^e \mathbf{1} \left(l_{i,t} \in g \right) + \delta_t + \delta_j + \delta_c + \varepsilon_{i,t}, \tag{36}$$

where δ_t, δ_j, and δ_c mark time, industry, and cohort of birth fixed effects. $e_{i,t}$ is the number of employees of age 45 years old and above, while $l_{i,t}$ is the number of employees overall. Indicator variables $\mathbf{1} \left(l_{i,t} \in g \right)$ are equal to unity if the total size of the establishment falls in bin g and zero otherwise. I estimate an equivalent regression for college-educated workers $s_{i,t}$.

Under what assumptions does the regression (36) identify the comparative advantage of experienced workers, $A_e(z, k)$? Intuitively, one can think about the identification in terms of the indirect inference. Equation (34) implies that in the model, in a given period t, the only reason for different firms to employ a different age composition of workers is the capital-experience complementarity, $A_e(z, k)$. Therefore, an estimate of $\beta_{g(z,k)}^e$ in equation (36) using the model-simulated data is exactly $\theta \log A_e(g(z, k))$, where $g(z, k)$ marks the size bin in which the establishments of type (z, k) falls. The indirect inference estimate of capital-experience complementarity requires choosing parameters $A_e(z, k)$ for all (z, k) such that $A_e(g(z, k)) = \exp \left(\hat{\beta}_{g(z,k)}^e \theta^{-1} \right)$. An analogous reasoning can be used to derive an estimate the capital-education complementarity $A_s(z, k)$.

This intuition illustrates that the following conditions need to hold to identify the productivity schedules:

i) elasticity of substitution between different types of labor, θ, is time-invariant.

ii) $\mathbb{E}(\varepsilon_{i,t}) = 0$ and $\mathbb{E}(\varepsilon_{i,t} \times \mathbf{1}(l_{i,t} \in g)) = 0$ for all i, all t, and all g. In other words, conditional on year, industry, and cohort of birth fixed effects, the only reason why larger firms hire more experienced and more educated workers is the time-invariant comparative advantage of those workers when working for more productive and more capital-intensive firms.

Simulated method of moments.

Given the productivity schedules $A_e(z, k)$ and $A_s(z, k)$, the remaining 10 parameters are estimated jointly by minimizing the equally weighted sum of the squared percentage deviations between 10 moments in the model and their empirical counterparts. Although the parameters are estimated jointly, some moments are particularly informative about specific parameters. The mean plant size helps identify the mean idiosyncratic productivity $\bar{\mu}_z$. The establishment size distribution provides information about the persistence of the idiosyncratic productivity shock ρ_z, its standard deviation σ_z, as well as the elasticity of substitution between the three types of labor θ. The economy-wide exit rate and the exit rate of plants of age 1 provide information about the mean and the standard deviation of the operating costs $\bar{\mu}_f$ and σ_f. The size distribution of entrants informs the entrant capital cost ζ_e and the initial productivity distribution ξ_q. In order to estimate the parameters of the capital adjustment cost, I use moments of investment rates based on the German USTAN dataset, a firm-level balance sheet data base compiled by the German central bank. The moments are directly calculated and reported in Bachmann and Bayer (2013). The skewness and kurtosis of the investment rates, together with the fraction of the investment rates exceeding 20%, are used to pin down the parameters of the capital adjustment cost ζ_0 and ζ_1. Table 2 presents the estimated values of the parameters together with the values of the targeted moments in the data and in the model.

4.3 Estimated Parameters and Model Fit

The right-hand side of Table 2 summarizes the model fit. Since there is a highly non-linear relationship between parameters and model moments, the match is not exact. Nevertheless, Table 2 reveals that the model matches well the size distribution of plants and their life-cycle dynamics. The estimated value of the persistence of the idiosyncratic shock is $\rho_z = 0.78$, in line with the literature.[21] The estimate of the standard deviation of the idiosyncratic shock $\sigma_z = 0.33$ is somewhat larger than typically estimated in the literature.[22] Without the ex-ante heterogeneity, a large dispersion of idiosyncratic shock is required to replicate the size distribution of plants in the data. The elasticity of substitution between the three types of labor is estimated at $\theta = 1.6$ within the range of the estimates in the literature.[23] The distribution of

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[21] Using German USTAN data Bachmann and Bayer (2014) estimate the persistence of the idiosyncratic productivity component at 0.9675. The estimates for the U.S. vary from 0.43 in Castro et al. (2015) to 0.8 in Foster et al. (2008).

[22] Using German data Bachmann and Bayer (2013) estimate the average standard deviation of the idiosyncratic risk at 0.095. The estimates for the U.S. industries reported in Castro et al. (2015) vary between 0.067 to 0.352

[23] The elasticity of substitution between college- and high-school-educated workers ranges from 1.4 in Katz and Murphy (1992) for the U.S. to 2.5 in Card and Lemieux (2001) who study the U.S., the U.K. and Canada. Card and

Table 2: Estimated Parameters and Model Fit

Parameter		Value	Target	Data	Model
I. Incumbent plants					
$\bar{\mu}_f$	mean of operation cost	17.86	exit rate of age-0 plant (%)	17.83	18.39
σ_f	st. dev. of operation cost	58.53	mean exit rate (%)	6.43	6.40
$\bar{\mu}_z$	mean productivity	2.35	mean size	14.66	14.69
σ_z	productivity st. dev.	0.33	share of plants of size 1000+ (%)	0.12	0.0070
ρ_z	productivity persistence	0.78	kurtosis of inv. rates	20.036	19.33
ζ_0	non-convex adj. cost	828.17	share of inv. rates above 20 (%)	13.80	14.61
ζ_1	convex adj. cost	920.35	skewness of inv. rates	2.192	3.04
θ	elast. of subst. between labor	1.60	share of firms 1-4 (%)	63.60	51.98
II. Entrants					
ξ_q	shape of init. productivity distr.	0.85	relative size of entrants (%)	19.44	21.13
ζ_e	entrant capital adj. cost	0.23	share of startups of size 100+ (%)	0.14	0.2773

Notes: The table presents the results of the SMM estimation. The three columns on the left show the parameters and their estimated values. The three columns on the right present the targeted moments and their values in the data and in the model. Moments related to the investment rates, skewness, kurtosis, and fraction of investment rates exceeding 20% are directly estimated using German data by Bachmann and Bayer (2013). The remaining targets are calculated using the BHP establishment panel.

investment rates exhibits positive skewness and large kurtosis, while the ratio of skewness to kurtosis is similar to the one in the data. Investment rates are lumpy, as captured by frequent spikes (the investment rates exceeding 20%). The wages of experienced and college-educated workers implied by market-clearing in the baseline calibration are consistent with estimates for Germany in the 1980s (see Fuchs-Schündeln et al. 2010). The experience wage premium equals 20% and the college wage premium 42%.

Figure 3 presents the shares of experienced (left panel) and educated (right panel) workers as a function of firm size. The blue solid lines present the shares in the model and the the red dashed line the shares in the BHP establishment panel. As a result of the production complementarities, the employment of experienced and educated workers is concentrated in the largest plants. The relationship between worker demographics and firm size is the key mechanism through which the labor force demographics interacts with business dynamism. The model matches the data very closely.

Figure 4 presents the life-cycle dynamics of the production units in the model. The blue solid line corresponds to the simulated data and the red dashed line to the BHP panel. All moments from the BHP panel are calculated using the sub-sample period from 1976 to 1985. Since the records begin in 1976, the oldest plant of known age is 10 years old in 1985. Panels (A) and (D) present how the exit rate varies with plant's age and size. Consistent with the data, the exit rate in the model tends to go down with plant's age and size. The shape and the magnitude of the exit rates resemble the rates in the data. It is evident from Panel (B) that

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Lemieux (2001) estimate the elasticity of substitution between workers of different age groups at 4 to 6.

Figure 3: The relationship between the plant size and employee demographics in the model and in the data.

(A) Experience share

(B) Education share
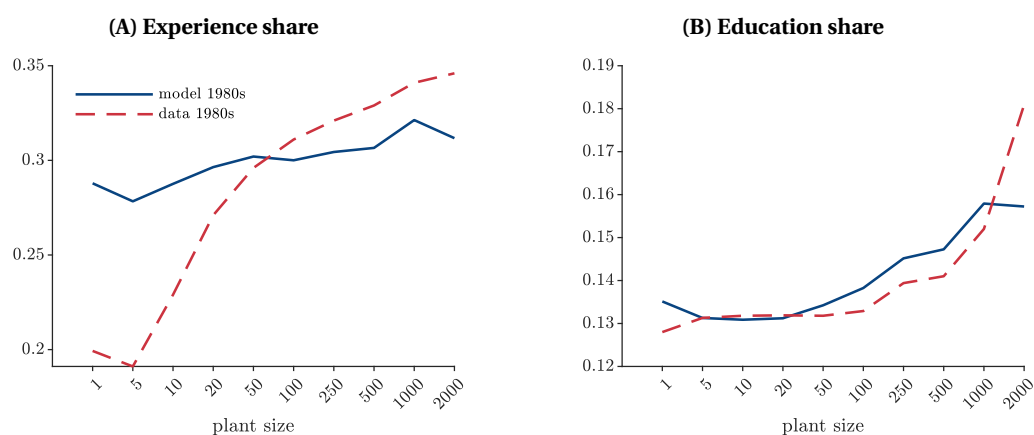
Notes: The blue solid lines present the share of the share of the experienced workers (left panel) or the share of the college-educated workers (right panel) in the model. The red dashed lines present the corresponding shares the BHP establishment panel in years 1976 - 1985.
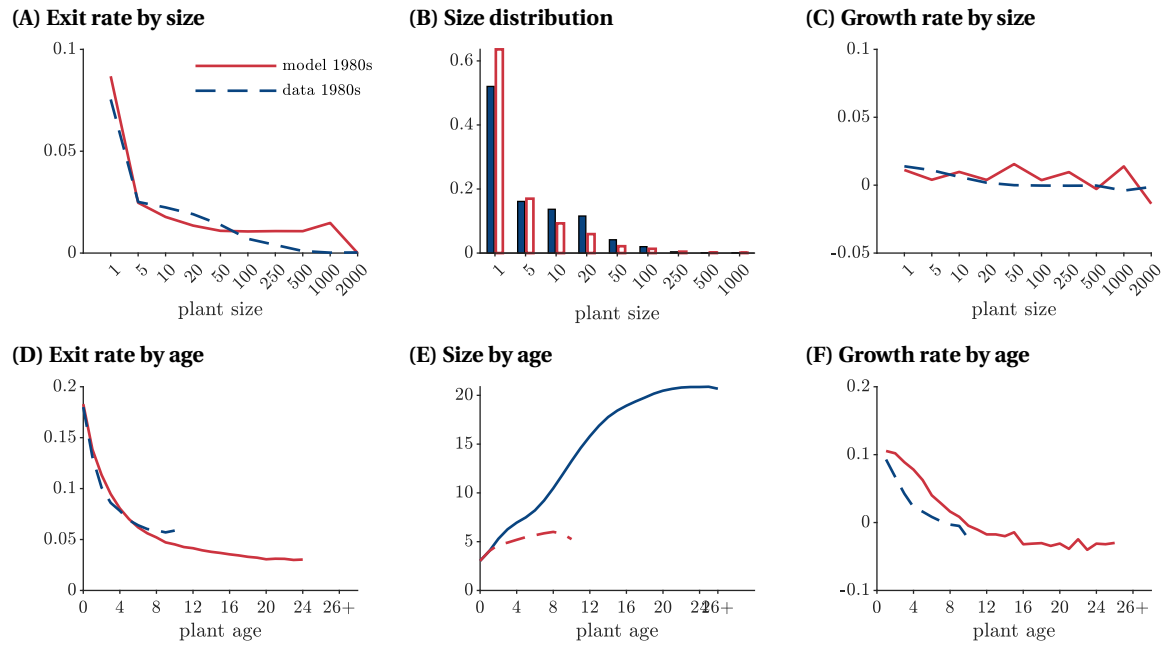
the size distribution of plants in the model matches well the size distribution of plants in the BHP panel. Consistent with the data, the distribution is highly skewed, with more than 50% of establishments having less than 4 employees. Furthermore, there is a considerable mass in the right tail: more than 40% of all workers are employed in plants larger than 100, even though these plants constitute only 2.5% of plants.

The establishment growth rates decline steadily as plants get older and larger (Panels E and F).[24] Young firms in the model exhibit more rapid growth as compared to young firms in the data. The underlying reason is that in the model there is no persistent heterogeneity that may play an important role in reality (see Pugsley et al. 2018). In the model, young firms are endowed with some productivity and initial capital level. Therefore, all differences between startups are transitory and the time of birth the majority of startups are below their optimal size. As they converge towards the optimal size, young plants exhibit rapid growth. However, in reality, some small, young plants may already be at their optimal size.

To the extent that the model over-predicts the growth rate of small and young plants, it may overstate the effect of demographic trends on the aggregate job creation rate. Nevertheless, the model reproduces qualitatively the life-cycle dynamics of establishments in Germany in the 1980s (Figure 4) Most importantly, it replicates the relationship between plant demographics and employee demographics, as shown in Figure 3.

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[24]This is also the case for U.S. firms. See Haltiwanger et al. (2013) for an overview.

Figure 4: Business dynamism in the model and in the data.

(A) Exit rate by size

(B) Size distribution

(C) Growth rate by size

(D) Exit rate by age

(E) Size by age

(F) Growth rate by age

Notes: The blue solid lines present the statistics implied by the model. The red dashed lines present the corresponding statstics in the BHP establishment panel estimated using the data from 1976 to 1985. The oldest plant of known age is 10 years old in 1985, hence the red dashed lines in Panels (D), (E), (F) do not extend beyond the age 10.

5 Impact of the Demographic Trends on Business Dynamism

In this section, I describe how I use the parameterized model to quantify the impact of the changes in the labor force composition on the size distribution of plants, firm entry, and job creation. As described earlier, the model replicates the West German economy in the period 1976 - 1985 (Section 4). By changing the parameters of household composition, I recreate the trends in the labor force observed between the 1980s and 2010s and study the new balanced growth equilibrium of the model.

I begin with an experiment in which I simultaneously change the structure of the labor force along three dimensions: the growth rate of the labor force, the relative supply of experienced labor, and the relative supply of college-educated workers. Next, I simulate the model changing only one dimension at a time. This allows to look into the mechanism driving the results and to assess the relative importance of the individual trends on business dynamism.

Solving for the new equilibrium.

In the model studied in the current paper, an increase in the relative supply of one labor type leads to a decrease in its price. However, despite the increase in the relative supply of experienced and college-educated workers, the data reveals that the price of experience has increased while the college wage premium has been stable (see Fuchs-Schündeln et al. 2010). Arguably, the prices of the two types of labor have been affected by the technological advancements biased towards experienced and educated labor.[25]

One may worry that the concurrent skill-biased changes in the technology invalidate the results presented in the current section. It is important to stress that the mechanism transmitting the changes in labor supply to the adjustments in the structure of labor demand does not hinge on the decline in the wage *level*. Rather, the results are driven by a fall in wages *relative* to the marginal productivity of labor. To demonstrate that the model accommodates technological changes, I design a model-consistent way to account for trends in productivity.

Recall from equation (34) that the share of experienced employees in a firm (z, k) follows

$$\log e(z, k) - \log l(z, k) = \theta \left(\log \bar{A}_e - \log w_e + \log A_e(z, k) \right)$$

and analogously in the case of demand for educated labor services. To find the new stationary equilibrium that corresponds to the new labor supply composition, I proceed as follows. I fix the returns to experience and the college wage premium at the values estimated in Fuchs-Schündeln et al. (2010) for the year 2009. I then proceed to solve for the price of raw labor w_l and technology parameters $\bar{A}_e$ and $\bar{A}_e$ that guarantee the clearing of the markets for the three types of labor in the new stationary equilibrium. The production complementarities, $A_e(z, k)$ and $A_s(z, k)$, are held fixed at the estimated values.
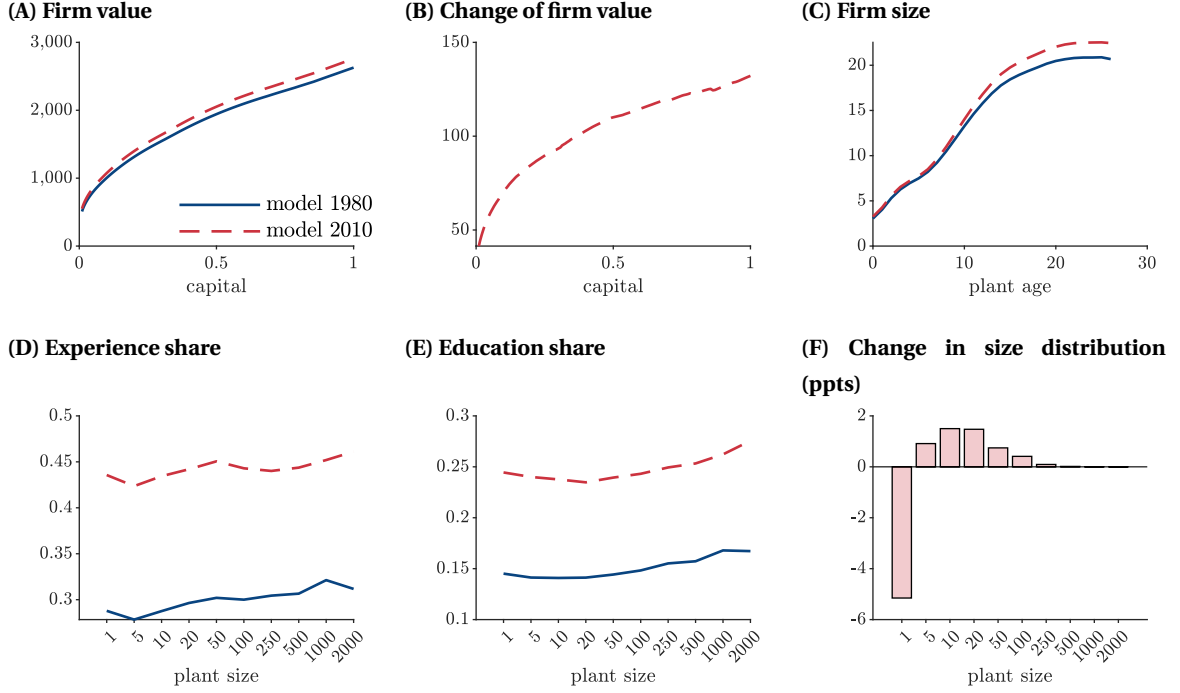
5.1 Impact of All Three Trends

To quantify the effects of the demographic trends on business dynamism, I perform the following experiment. I replicate the developments in the German labor market between the 1980s and the 2010s. I change the value of the parameter describing the population growth rate from 0.64% to 0.43%. I increase the share of college-educated workers by 73%, from 15% to 26%, and the share of experienced workers by 46%, from 30% to 44%. Afterwards, I analyze the effects of these changes on the balanced growth path equilibrium. I solve for the new price of raw labor w_l, price of experience w_e, and price of skills w_s that clear the markets for the three types of labor.

The first set of results is presented in Figure 5. The black solid lines present the baseline simulation (the model's parameters reflect the German labor force in the 1980s), while the red dashed lines present the results under the new demographic structure. All plants benefit from the increase in the relative supply of experience and education, since these two types of labor are relatively more productive. Panel (A) in Figure 5 presents the mean firm value function for each admissible level of capital $\mathbb{E}_z V(z, k)$. The value function shifts upwards.

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[25]Acemoglu (2002) and Acemoglu and Autor (2011) present an overview of evidence for the skill-biased technological change, while Caselli (2015) argues that the recent technological progress has also been biased towards experience.

Figure 5: Macroeconomic impact of the three secular demographics trends.

(A) Firm value

(B) Change of firm value

(C) Firm size

(D) Experience share

(E) Education share

(F) Change in size distribution (ppts)

Notes: The blue solid lines present statistics from the model simualtion in the initial balanced growth equilibrium. The red dashed lines correspond to the outcomes under the new labor force composition. A general equilibrium response.

Importantly, these gains are more pronounced among the largest plants. To illustrate this point, in Panel (B) I plot the difference between $\mathbb{E}_z V(z,k)$ in the new equilibrium and in the baseline. The difference is increasing in capital. The reason is that the increase in the supply of experienced and skilled labor entails a fall in the corresponding wages. However, firms do not equally benefit from the falling wages. Recall that the unit cost of the labor composite is the weighted average of the wages of the three types of labor, with weights given by the productivity of each type of labor in a plant (z,k)

$$W(z,k) = w_l \left[1 + A_e(z,k)^\theta \omega_e^{1-\theta} + A_s(z,k)^\theta \omega_s^{1-\theta} \right]^{-\frac{1}{\theta-1}}. \tag{37}$$

Note that experienced and skilled labor is more productive in large plants, as $A_e(z,k)$ and $A_s(z,k)$ are increasing in z and k. Therefore, the fall in wages of experienced and educated workers brings the largest reduction in the labor costs in highly productive and capital-rich plants.

As a results of these changes in the firm value, the the size profile of plants over the life-cycle shifts upwards - plants of any age are now larger – as revealed in Panel (C) in Figure 5. The effect is more pronounced for the oldest enterprises since they rely heavily on experience

Figure 6: Out-of-sample Predictions of the Model.

(A) Education share **(B) Experience share** **(C) Change in size distribution (ppts)**
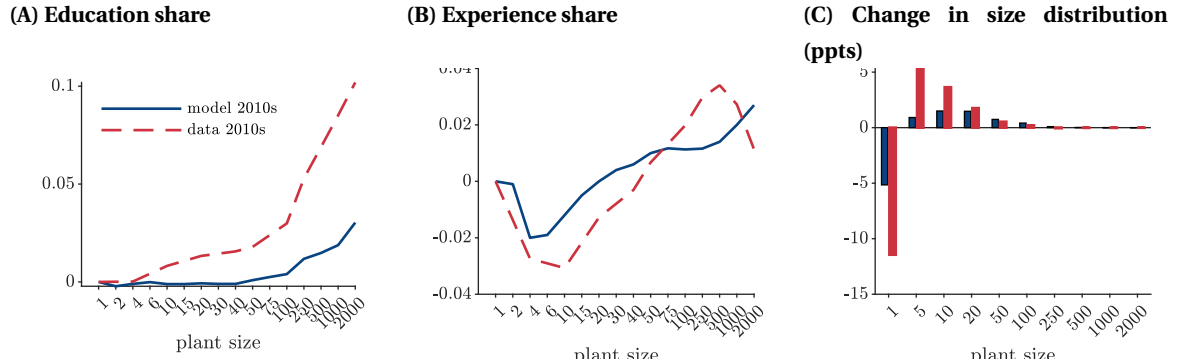
Notes: The blue solid lines present statistics from the model under the labor force composition as in the 2010s. The red dashed lines present corresponding outcomes in the data.

and skills. This result is consistent with the empirical trends documented in Figure 1 in Panel (B).

In Panel (F) I present the change (in percentage points) in the size distribution of plants. We can see that the fraction of production units employing less than 5 workers drops significantly, by 5 percentage points, while the mass shifts to the right as the number of plants in the remaining size bins increases. There are two forces driving the changes in the size distribution of plants. Firstly, all plants employ relatively more experienced and skilled workers (Panels D and E). These workers offer a type of labor complementary to capital, so the marginal product of capital rises. In consequence, all plants accumulate more capital and grow in size. Secondly, the in the new equilibrium there is a lower startup rate. Since older plants tend to be larger, this change in the number of entrants entails an increase in the average size of production units.

Figure 6 presents the out-of-sample predictions of the model. We see that the share of skilled workers (Panel A) and the share of experienced workers (Panel B) shift upwards in all size bins. Qualitatively, it is consistent with the model's predictions. Quantitatively, however, the model underestimates the shift of the demand towards skilled labor. Moreover, the model predicts a much higher share of experienced workers in small firms and a larger increase in the share of skilled workers in large firms. Furthermore, the model predicts the large drop in the percentage of small firms and the increase in the share of firms in the remaining size bins, as in the data (Panel C). This suggests that the shape complementarities between skilled labor and physical capital have changed over the last 4 decades. The impact of these changes on firm dynamics constitutes a promising avenue for future research.

Quantitatively, the three demographic trends can fully account for the increase in plant size and the decline in business creation we observe in the data. The results are summarized in Table 3. In the model, the average plant size increases from 14.69 to 17.13 employees, a little more than in the data. The three demographic trends account for 65% of the decline in the startup rate observed in the data. Since in the balanced growth equilibrium the exit rate

Table 3: Effects of the Demographic Trends on Business Dynamism

| | 1976-85 | | 2008-17 | | |
	data	model	data	model	$\frac{\Delta \text{ model}}{\Delta \text{ data}}$
I. Business Dynamism					
plant size	14.66	14.69	16.88	17.13	109.26%
mean growth	1.04	9.23	0.80	8.10	52.75%
job creation	14.12	17.82	13.03	17.68	10.12%
job destruction	13.60	9.72	12.80	9.79	-10.95%
startup rate	5.24	6.99	3.21	5.24	64.52%
II. Concentration					
share 100+	2.07	2.48	2.49	2.91	85.05%
III. Aggregate Outcomes					
GDP per capita	1.00	1.00	1.71	1.54	75.65%

Notes: Selected measures of business dynamism. The first two columns correspond to the initial balanced growth equilibrium, the following two columns report moments under the new structure of the labor force. The "data" columns present the moments in the BHP panel. The "model" columns present the moments implied by the model simulation. The last column tells us the what fraction of the change in the data is explained by the model. I normalize the real GDP per capita in the period 1976-1985 to unity both in the model and in the data (the actual values are taken from the World Bank database).

equals the startup rate net of the growth rate of the labor force, the model predicts a large drop in the exit rate. This contrasts with the data, as the observed mean exit rate declines only slightly (from 5.8% to 5.6%).

Recall that large and old firms tend to grow at a slower pace and create less jobs on average (see Panels C and F in Figure 4). A drop in the startup rate results in an increase in the average firm age which leads to fall in the job creation. The results indicate that the demographic trends can account for half of the decline in the average growth rate and 10% of the decline in the aggregate job creation rate. On the other hand, the model predicts an increase in the job destruction rate, but this is not what we see in the data. Finally, the model accounts for 85% of the increase in concentration, measured by a share of plants larger than 100 employees. Since experienced and educated workers are more productive, the model also predicts a significant increase in the real GDP per capita.

5.2 Understanding the Mechanism

This section describes the effects of each of the three demographic trends in isolation. This allows me to shed light on the mechanism underlying the interactions between heterogeneous

plants and heterogeneous workers.

Slowdown of the population growth rate.

I begin with an experiment in which the relative supply of experience and education is held fixed, while the growth rate of the labor force g_n declines from 0.64% to 0.43%.[26] All other parameters of the model are kept at their estimated values.

Firstly, I study the partial equilibrium response in which I fix the wages at the values corresponding to the levels observed in Germany in the 1980s. I will relax this assumption later on, in the general equilibrium experiment. I solve for the new stationary firm measure per capita $\hat{\mu}$ implied by the lower population growth rate and the policy functions corresponding to the prices from the baseline calibration.

In response to the slowdown in the population growth rate, the startup rate decreases by 0.43% percentage points, from 6.99% to 6.56%. Along the balanced growth path the startup rate equals the aggregate exit rate plus the growth rate of the labor force, hence the following two forces are driving the decline in the startup rate in the model. Firstly, for a fixed aggregate exit rate, the startup rate declines by the amount equal to the difference in the population growth rates between the 1980s and 2010s, $0.64\% - 0.43\% = 0.21\%$. Furthermore, the firm distribution shifts towards larger establishments which are less likely to exit. The aggregate exit rate declines, leading to a proportional drop in the startup rate.

Since entrants tend to be smaller than the incumbent plants, the drop in the startup rate causes an increase in the mean plant size from 14.69 to 15.60 employees. As older production units tend to be closer to their optimal size, they do not create as many jobs as young firms, hence the average job creation rate declines.
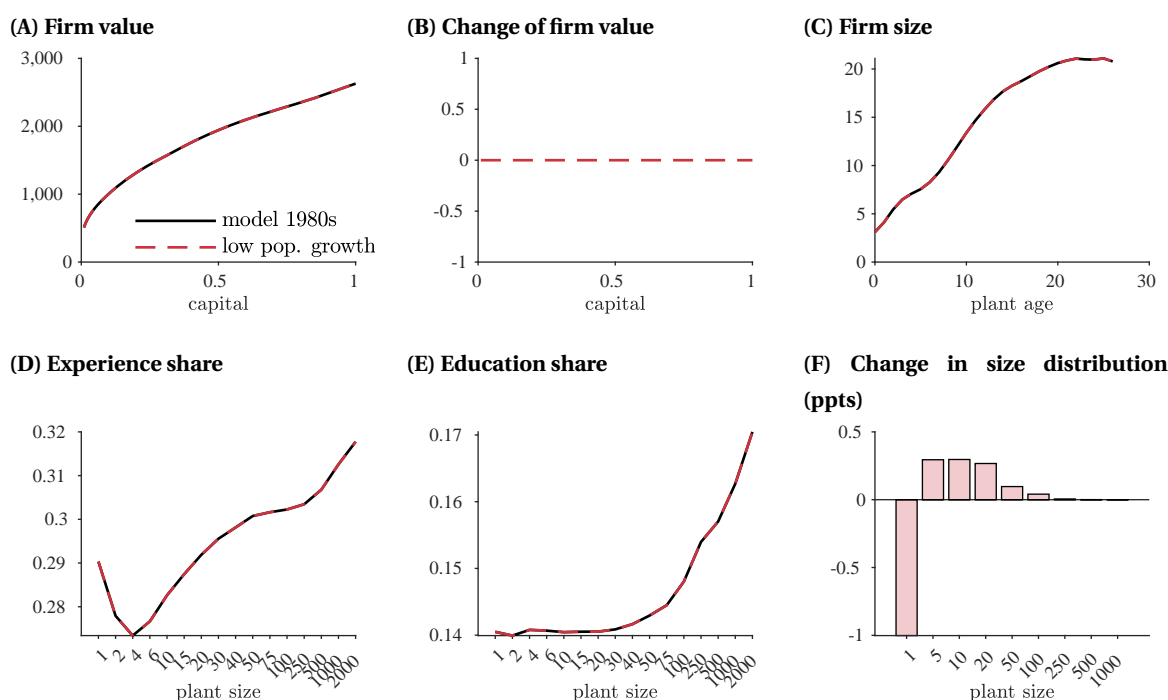
Figure 7 presents the results of this partial equilibrium experiment. As depicted in Panel (F), the size distribution of production units shifts towards larger entities. Since the wages are fixed, the firm value (depicted in Panel A) and the relative employment of experience and education (Panels D and E) remain the same as in the baseline.

The partial equilibrium effects in the current model are similar to the results in Hopenhayn et al. (2018) and Karahan et al. (2018). Using a model with homogenous labor force, the authors show that the economy adjusts to the lower growth rate of the population entirely through changes in the entry rate, while prices remain intact. This corresponds to the partial equilibrium studied above.

However, the production complementarities in the current model generate novel general equilibrium effects. The availability of skilled and experienced labor puts a constraint on the size distribution of plants. To see this, I consider the following general equilibrium experiment. Again, I start with changing the population growth rate from 0.64% to 0.43%. However this time, in contrast to the previous exercise, I allow wages to adjust to clear the markets for the three types of labor. At the same time I keep the relative supply of experience and education fixed at the level of the baseline calibration. This may seem counterintuitive since, in

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[26]In the U.S. the decline in the growth rate of the labor force over this time period is significantly more severe. As reported in Hopenhayn et al. (2018), in the U.S the labor force growth dropped from 1.64% in 1980s to 0.42% in 2010s.

Figure 7: Slowdown in the population growth rate in partial equilibrium.

(A) Firm value

(B) Change of firm value

(C) Firm size

(D) Experience share

(E) Education share

(F) Change in size distribution (ppts)

Notes: The black solid lines present the statistics from the model simualtion in the 1980s. The red dashed line corresponds to the model outcomes under the lower population growth rate.

reality, the slowdown in the growth rate of the labor force tends to lead to an older workforce. Nevertheless, I consider this hypothetical scenario to better illustrate the mechanism.
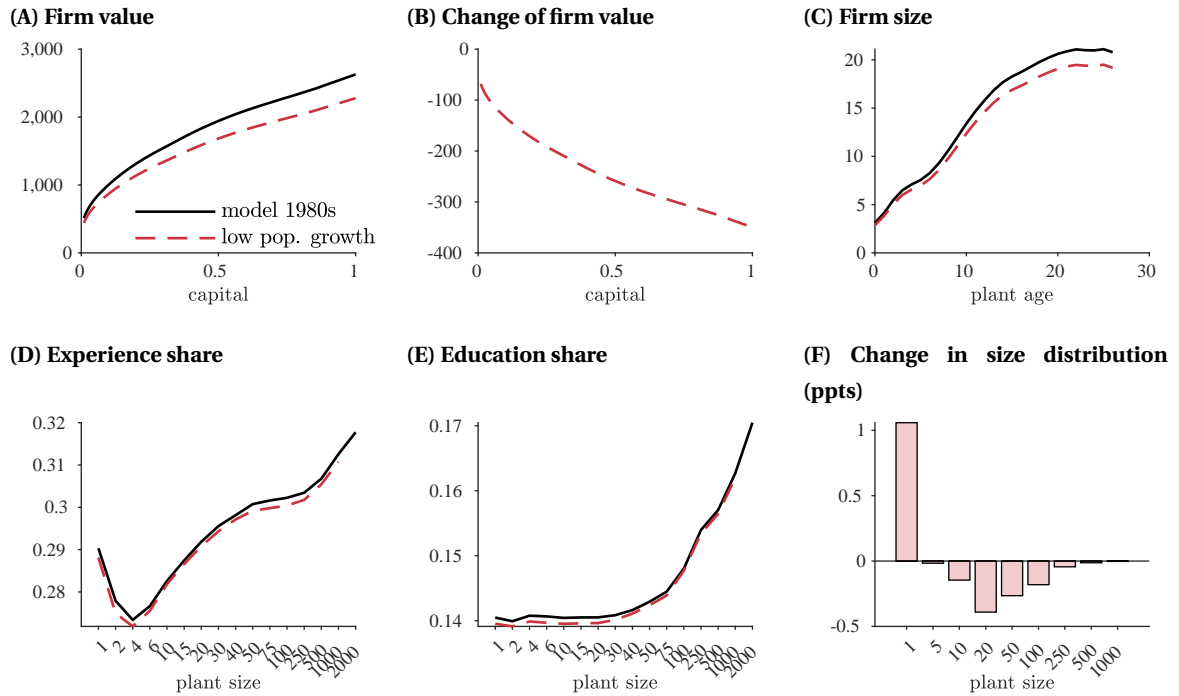
The economy adjusts to the new demographic structure in the following way. As explained above, the slowdown in the population growth rate implies a shift in the size distribution of plants towards larger units. Due to the production complementarities, a greater number of large firms entails a higher aggregate demand for experience and skills. Since the relative supply of the two types of labor is fixed, the returns to experience and the college wage premium have to rise to clear the labor market. The experience premium increases from 20% to 26% and the college wage premium from 40% to 44%.[27]

Affected by the rising prices of experience and skills, capital-rich establishments shrink. Young plants tend to have less capital and rely on raw labor that becomes relatively less expensive. Consequently, the life-cycle size profile shifts downwards and the size distribution of establishments tilts towards smaller units. These results are presented in Figure 8.

The model predicts a reduction in the average size of plants from 14.69 to 13.80 employees. In the general equilibrium, the startup rate declines slightly less than in partial equi-

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[27]Fuchs-Schündeln et al. (2010) report a stable university wage premium in Germany between 1985 and 2004, and an increase in the price of experience from 20% to 40% in that period.

Figure 8: Slowdown in the population growth rate in the general equilibrium.

(A) Firm value

(B) Change of firm value

(C) Firm size

(D) Experience share

(E) Education share

(F) Change in size distribution (ppts)

Notes: The black solid lines present statistics from the model simualtion in the initial balanced growth equilibrium. The red dashed line corresponds to the model outcomes under lower population growth rate.

librium, from 6.99% to 6.65%. The direct effect of the lower growth rate of the labor force is partially offset by the indirect effect that stems from the reduction in plant size. Smaller units record lower profits and are more likely to leave the market. The aggregate exit rate increases and pushes upwards the rate of firm creation.

The above experiment illustrates that unless the slowdown in the growth rate of the labor force is accompanied by a sufficient increase in the stock of human capital, the life-cycle size profile of plants shifts downwards in response to the slowdown in the population growth. The patterns observed in the data seem to be consistent with the above reasoning. The average plant size conditional on age has declined in the U.S. (see Hopenhayn et al. 2018), in contrast to Germany. At the same time, the slowdown in the growth rate of the labor force has been much more severe in the U.S., making the induced shift towards older, larger firms more pronounced. However, the trends in the share of experienced and skilled workers are quantitatively similar. Interpreted through the lens of the current model, the decline in the average firm size conditional on age observed in the U.S., might be a result of an insufficient supply of skilled and experienced workers in the U.S. labor market.

Table 4: Impact of the changes in the demographic structure of the labor force

| | 1976-85 | | 2008-17 | | | |
	data	model	low g_n	education & aging	all	data
I. Business Dynamism						
plant size	14.66	14.69	13.80	16.92	17.13	16.88
mean growth	1.04	9.23	9.28	8.03	8.10	0.80
job creation	14.12	17.82	17.83	17.67	17.68	13.03
job destruction	13.60	9.72	9.72	9.79	9.79	12.80
startup rate	5.24	6.99	6.65	5.54	5.24	3.21
II. Concentration						
share 100+	2.07	2.48	2.17	2.95	2.91	2.49
III. Aggregate Outcomes						
GDP per capita	1.00	1.00	1.03	1.40	1.54	1.71

Notes: Table presents selected measures of business dynamism for all quantitative experiments. The columns labeled "data" correspond to moments calculated using the BHP establishment panel. All results include general equilibrium effects.

Increase in the share of experienced and college-educated workers.

Let us now consider the macroeconomic impact of an exogenous rise in the relative supply of experienced and college-educated workers. I increase the share of experienced workers from 30% to 44% and the share of skilled workers from 15% to 26%. I solve for the new wages of raw labor as well as the new skill-biased technology parameters to clear the labor market.

In response to changes in supply, the relative wages of experienced and skilled workers fall. The demand curve for the two types of labor shifts upwards. Due to the production complementarities, capital becomes more productive and firms accumulate more of it. The equilibrium distribution of firms shifts towards larger units.The quantitative results, summarized in Table 4, are very similar to the experiment in which all three demographic trends are considered jointly. We can see that almost the entire change in the size distribution of plants in Germany is driven by the trends in the supply of experience and skills. The slowdown in the growth rate of the population plays only a minor role.

The importance of plant heterogeneity.

In this section, I decompose the changes in the aggregate variables into direct and indirect effects. The two effects are defined in the following way. As I explained in the previous sec-

Table 5: Percent of the aggregate change due to the indirect effect.

experiment	mean size	exit rate	GDP p.c.	demand for experience	skills
all trends	62.80%	49.77%	115.24%	0.09%	-0.03%
lower pop. growth	-23.07%	116.78%	-2499.91%	0.66%	2.52%
education & aging	56.82%	41.88%	120.86%	0.09%	-0.01%

Notes: Decomposition of changes in selected measures of business dynamism into direct and indirect effect. I define the direct effect as the changes implied by the new policy functions, keeping constant the number of plants of each type. The indirect effect captures the changes stemming from the shift in the equilibrium distribution of plants $\hat{\mu}(z, k)$.

tions, in response to the fall in wages, plants of all types (z, k) change their policy functions. I will call this the *direct effect*. In other words, the direct effect describes the changes in aggregate outcomes if there was no shift in the size distribution of firms. However, the important implication of my model is that the changes in wages lead to a shift in the equilibrium plant distribution and the following changes in aggregates, all of which I call the *indirect effect*. The indirect effect captures the changes in the aggregate outcomes stemming from differences in the number of firms of type (z, k), keeping the behavior of firms constant.

Table 5 presents the relative importance of the indirect effect for selected aggregate statistics. Each row corresponds to one of the general-equilibrium experiments discussed in the previous sections. In the first row I consider all three trends (see Section 5.1). The following rows correspond to the experiment in which I only adjust the population growth rate (the second row), and supply of experience and education (the third row). Comparing the effects in the three experiments, the indirect effect accounts for around 60% of changes in the mean size of establishments. It is also responsible for 42% to 50% of the adjustments in the aggregate exit rate. The magnitude of the indirect effect is notable, and it demonstrates the importance of modelling explicitly the heterogeneity in the production side of the economy.

Moreover, the indirect effect accounts for more than 100% of the change in GDP per capita, meaning that the direct effect is negative. The negative sign of the direct effect hinges on the assumption of decreasing returns to scale, which implies that a large number of small firms generates output that is *higher* than the output generated by a small number of large firms (keeping the number of workers fixed). In this case, the entire increase in the GDP per capita is due to the indirect effect: the shift in the plant distribution towards entities that use workers' human capital more effectively. The opposite is true in the experiment altering the population growth rate (the second row in Table 5): firms get smaller and, all other things equal, more productive.

It is worth stressing that my results differ from the predictions of the homogenous-labor model studied in Hopenhayn et al. (2018) and Karahan et al. (2018). In their model, the demographic change is accommodated fully by the entry margin, while the equilibrium prices and

the firm's policy functions remain intact. Using the terminology introduced in this section, the indirect channel is the only channel of adjustment. However, I argue that in the presence of the production complementarities, changes in the supply of human capital entail a revision of the firm's strategy. The model implies that plants born in later cohorts will be larger at any given age. This is indeed the case in Germany as documented in Panel (A) in Figure 1.

Interestingly, virtually none of the increase in the aggregate demand for experienced and educated labor results from the indirect effect. To understand why, recall the results presented in Panels (E) and (F) in Figure 5. A larger supply of experience and skills is accommodated by the parallel shift of the labor demand curve for all plants. The changes in the equilibrium distribution of plants become inconsequential.[28]

In conclusion, the estimated production complementarities imply that the changes in the demographic structure of the labor force have a large macroeconomic impact. The consequences of the rising supply of skilled and experienced labor include a higher average size of production units, a lower startup and exit rates, and a slower pace of worker reallocation.

6 Reduced-form Evidence for the Model's Predictions

The model developed in the current paper predicts that in the economy characterized by an older and more educated labor force, the number of business startups is lower, while economic activity is more concentrated in large firms. In this section, I provide a reduced-form evidence that the relationship predicted by the model holds across industries in Germany.

To this end, I aggregate the establishment-level data in the BHP panel into 301 industries using the 3-digit classification code. As for the measures of business dynamism, I consider five different characteristics of the industry: the average plant size, share of workers employed in plants larger than 100 employees, share of plants of age 11 or older (which I call mature), share of plants of age 0 (which I call startups), and share of workers employed in the 5 largest plants. I calculate these measures of each industry in each year and test the relationship between the business dynamism and the relative intensity of using experienced and educated labor. I estimate a set of regressions in the following form

$$\log\left(y_{i,t}\right) = \alpha_1 + \alpha_2 \log\left(E_{i,t}\right) + \alpha_3 \log\left(S_{i,t}\right) + \zeta_i + \zeta_t + \varepsilon_{i,t}, \tag{38}$$

where $y_{i,t}$ is a measure of business dynamism in industry i in year t, whereas $E_{i,t}$ marks the percent of experienced workers into the total number of employees in a given industry. The independent variable $S_{i,t}$ is defined as the percent of college-educated workers into the total number of employees in a given industry. I include time and industry fixed effects, ζ_t and ζ_i.

Firstly, I estimate (38) using OLS. However, the correlations between the industry characteristics and demographics of its employees may reflect common underlying factors that lead both to lower business dynamism and more intensive use of human capital. To alleviate these concerns, I use the aggregate shares experienced and skilled labor as an instrument for

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[28]In some cases the contribution of the indirect effect is negative. This is driven by an increase in the mass of plants in the firm size distribution where the demand for skilled labor becomes a downward-slopping function of size. The latter comes from the substitution between different types of labor.

the industry-level shares. I build on Nekarda and Ramey (2011) who identify industry-level effects of the aggregate changes in government spending. The identification is based on the fact that different industries are differently exposed to changes in the aggregate supply of experienced and skilled labor.[29] Therefore, I also estimate regressions (38) using two-stage least squares. In the first stage, the share of experienced (educated) workers in a given industry is projected on the share of experienced (educated) workers in the aggregate economy, allowing for the elasticity to vary across industries. That is, I estimate

$$\hat{E}_{i,t} = \beta_0 + \beta_i \bar{E}_t + \zeta_i + \zeta_t + \varepsilon_{i,t},$$
$$\hat{S}_{i,t} = \gamma_0 + \gamma_i \bar{S}_t + \xi_i + \xi_t + \nu_{i,t}, \tag{39}$$

where $\bar{E}_t$ and $\bar{S}_t$ mark the shares of experienced and educated worker in the whole economy, ζ_i, ξ_i denote industry fixed effects and ζ_t, ξ_t time fixed effects. In the second stage, I estimate regressions (38) in which I instrument experience and education shares with the values predicted from (39).

The five measures of business dynamism define the five different specifications presented in columns (1)-(5) in Table 6 The table presents the 2SLS estimates, while the OLS results are similar and presented in Appendix A.1.

Table 6: Employee demographics and business dynamism across German industries.

	(1) size	(2) 100+ share	(3) mature share	(4) startup share	(5) top 5 share
experience share	0.196***	0.523***	0.0566	-0.147*	-0.0780*
	(0.0912)	(0.0593)	(0.0368)	(0.0792)	(0.0433)
education share	0.575***	0.802***	0.0342	0.0977	0.720***
	(0.106)	(0.107)	(0.0495)	(0.108)	(0.0799)
Observations	12774	11181	12749	10674	12774
Adjusted R^2	0.941	0.921	0.673	0.610	0.947

t statistics in parentheses
* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Notes: The table presents the results of regressions (38) for 301 industries in Germany and all years 1976 - 2017. Five columns correspond to five different different measures of business dynamism: mean size, the share of employees in plants larger than 100 employees, share of plants older than 11 years, the share of plants of age 0, and the employment share of the top 5 largest plants. The independent variables include experience share (first row) and education share (second row).

German industries that were more exposed to the increase in the aggregate share of experienced and educated workers tend to exhibit larger average plant size, are characterized by a lower rate of entry and are more concentrated. These empirical results are consistent

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[29]The intuition behind the identification is similar to the Bartik shift-share instrument named after Bartik (1991).

with the predictions of the model and quantitatively significant. For instance, a one-percent increase in the share of skilled workers translates on average into a one-half percent rise in the average plant size.

In Appendix A.2, I show that across OECD countries there are similar correlations between the share of experienced and educated workers and the size distribution of manufacturing firms.

7   Conclusion

This paper develops and empirically validates a novel theory in which the composition of the labor force interacts with the life-cycle dynamics of firms. The interaction rests on complementarities in production between firms' capital and workers' experience and education. I estimate the complementarities using linked employer-employee data from Germany. The results demonstrate that changes in the structure of the German labor force between the 1980s and the 2010s can account for the observed reallocation of production towards larger, older, and less dynamic businesses. Consistent with the model mechanism, German industries more exposed to the secular trends in labor force composition, tend to exhibit larger plant size, higher concentration, and lower dynamism.

My results prove that the demographic structure of the labor force determines the types of firms operating in the market. In conclusion, to fully assess the macro-economic impact of demographic trends, it is crucial to account for the interactions between worker and firm heterogeneity.

A   Further Empirical Results

In this section I provide further empirical support for the main prediction of the model. Secondly, I document the cross-country correlations between the labor force composition and business dynamism in a sample of OECD countries. Finally, I show that the discussed relationships between firm's size and workers' human capital hold, even conditional on additional characteristics of workers, such age, occupation and cohort of birth.

A.1   OLS Results of Cross-industry Regressions

In this section, I present results of regressions (38) estimated by OLS. The results are qualitatively similar to 2SLS. Notable exception is positive relationship between the startup rate and share of skilled workers. This suggest that dynamic sector with large number of startups attracts skilled workers.

Table 7: Employee demographics and business dynamism across German industries.

	(1) size	(2) 100+ share	(3) mature share	(4) startup share	(5) top 5 share
experience share	0.228***	0.313***	0.116***	-0.283***	-0.0152***
education share	-0.143	0.458***	0.0869	0.343***	0.672***
Observations	12774	11181	12749	10674	12774
Adjusted R^2	0.940	0.918	0.677	0.610	0.948

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Notes: The table presents the OLS estimates of regressions (38). Columns (1) - (5) correspond to five endogenous variables which refer to different measures of business dynamism: mean plant size, the employment share of plants larger than 100 employees, the share of plants older than 11 years, the share of plants of age 0, and the employment share of the top 5 largest plants. The independent variables include experience share (first row) and education share (second row). The shares are instrumented with the variable corresponding to the exposure of the industry to the changes in the aggregate changes in labor supply.

A.2   Worker and Firm Demographics: Cross-country Correlations

In this section I present the cross-country correlations between the size distribution of firms in a given country and demographics of the labor force. I use OECD Structural and Demographic Business Statistics database that covers 27 countries.[30] Each dot in Figure 9 represents one OECD country in year 2016. Panel (A) presents the log of the mean firm size calculated by dividing the total number of employees by the total number of firms. Panel (B) presents the fraction of firms exceeding 250 employees relative to the total number of firms.

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[30]These countries are AUT, BEL, CZE, DEU, DNK, ESP, EST, FIN, FRA, GBR, GRC, HUN, IRL, ISL, ISR, ITA, JPN, LTU, LVA, NLD, NOR, POL, PRT, SVK, SVN, SWE, TUR. The dataset covers manufacturing firms only.
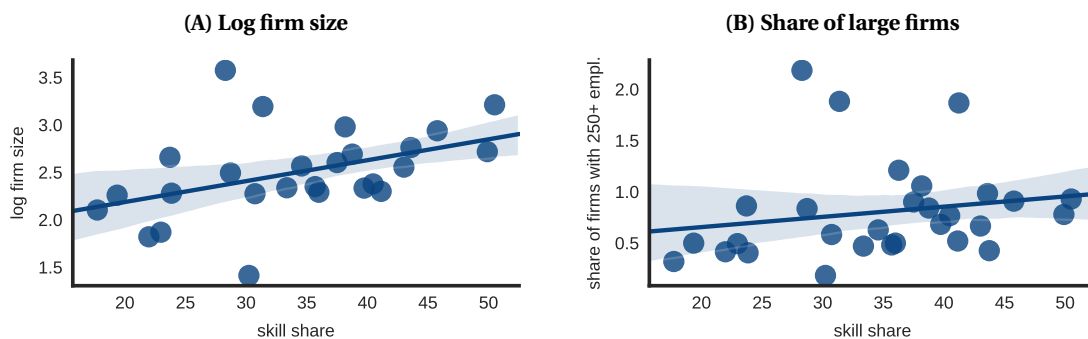
The blue solid line depicts the regression line and the shaded gray area marks the 95% confidence interval. There is a positive correlation between the share of the experienced workers in the labor force and the average firm size and the share of large firms in the economy. Figure 10 shows that there is a similar, albeit weaker, relationship between the share of the college-educated workers and the firm size distribution (unconditionally).

Figure 9: Experienced workers and business dynamism.
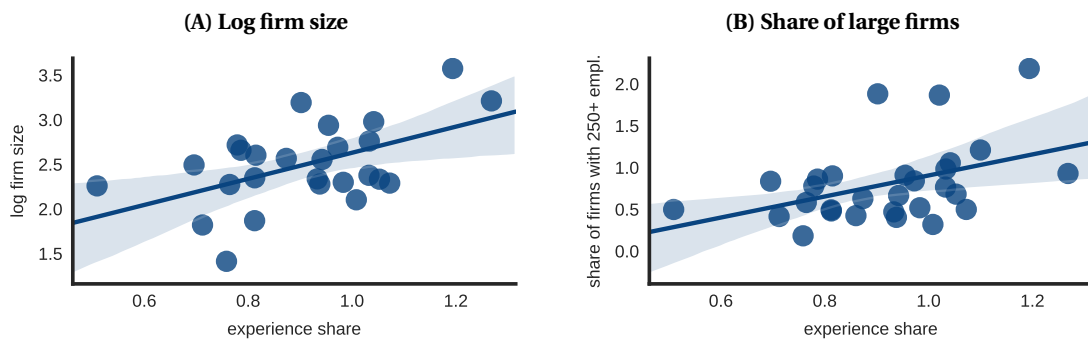
Notes: Data from OECD Structural and Demographic Business Statistics. Each dot represents a country in 2016. The blue solid line denotes the regression line and the shaded gray area marks the 95% confidence interval. The dataset covers manufacturing firms only.

Figure 10: College-educated workers and business dynamism.

Notes: Data by OECD Structural and Demographic Business Statistics. Each dot represents a country in 2016. The blue solid line denotes the regression line and the shaded gray area marks the 95% confidence interval. The dataset covers manufacturing firms only.

Table 8 presents the results of the linear regressions in which log of the average firm size (left column) and share of firms larger than 250 employees (right column) is projected on the size the share of experienced and the share of the college-educated workers.

The results suggest that the relationship between the supply of skills and business dynamism predicted by the model holds across OECD countries. Although the presented ev-

Table 8: Labor force demographics and business dynamism in OECD countries

	(1) log firm size	(2) share of large firms
experience share	0.0329*** (0.00812)	0.000574*** (0.000144)
skill share	0.0128*** (0.00368)	-0.000107 (0.0000653)
Observations	252	252
Adjusted R^2	0.174	0.053

Standard errors in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Notes: The table presents cross-country correlations between the structure of the labor force and firm size and a measure of concentration. The table reports coefficients in linear regressions in which average firm size and fraction of firms larger than 250 employees is projected on the fraction of workers 45 years or older (experience share) and fraction of workers with college degree or above (skill share). The estimates use pooled data for all countries and all years available in the OECD database.

idence is not sufficient for a causal interpretation, it provides a further support for model's key predictions.

A.3 Worker and Plant Demographics

In this section I use a more detailed data source to alleviate the concern that the relationship between plant size and employee demographics is spurious. Specifically, I show that the relationship between age, education, and employer size holds also conditional on additional workers' characteristics.

Linked-Employer-Employee-Data of the IAB (LIAB). In order to investigate in more detail the relationship between firm and worker demographics, I use the LIAB data set which combines establishment information from the Establishment History Panel (BHP) with a detailed employment biographies of individuals from an additional survey. The dataset contains detailed biographical information of all employees for a 2% sample of selected establishments surveyed in the BHP panel.[31] The information about the workers includes age, education, and occupation. Crucially, the data contains a direct measure of experience: a number of days of recorded employment. The data spans between 1994 and 2014.[32]

I use the LIAB data to study the relationship between employee demographics and the employer size in greater detail. I estimate a set of regressions in which the log size of the

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[31]Establishments included in LIAB tend to be much older and larger than the population. The mean size of the establishment in LIAB is 99.05 employees (15.59 in BHP). There are no plants younger than 28 years old.

[32]For more details on the data set and its construction see Klosterhuber et al. (2016).

employer is projected on employee education, age and experience. In the regressions I add the following fixed effects: worker's occupation and birth cohort as well as plant's age, cohort, and industry. Table 9 presents the results. The left column shows the coefficients of a dummy variable indicating whether the employee is older than 45 years, and a dummy indicating whether she has a college degree. Older individuals tend to work in 7.73% larger production units as compared to the younger workers (the mean and the standard deviation of the plant size is 99.05 and 553 employees, respectively). Having a college degree is associated with working in a 1.73% larger plant. The overall effect is small, but highly significant. Note that the magnitude of coefficient is influenced by inclusion of the control variables, including fixed effects for worker's occupation and birth cohort, as well as plant's industry, cohort, and age.

The right column in Table 9 presents the results of a regression in which the experienced measured as the number of years in employment (rather than proxied by age). Again, individuals with more experience tend to work in larger plants.

Table 9: The employment of skilled labor and business dynamism across German industries.

	(1) log plant size	(2) log plant size
age ≥ 45	0.0773*** (0.000825)	
experience		0.0147*** (0.000003)
experience^2		0.0004*** (0.000001)
university degree	0.0173*** (0.0006)	0.0648*** (0.0006)
worker and plant FE	yes	yes
Observations	47928986	47928986
Adjusted R^2	0.316	0.321

Standard errors in parentheses
* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Notes: Table presents the results of linear regressions in which employer size is projected on employee characteristics. Left column corresponds to a regression in which is proxied by age, while the right column to a regression in which experience is measured as a number of years a given worker spent in employment. The first column corresponding to a coefficient of a discrete variable attaining one if individual is weakly older than 45 years. The second (third) row corresponds to a continous variable measuring number of years of recorded employment spell (squared). The fourth row corresponding to a discrete variable attaining one if the individual has a university degree. Regressions control for the following fixed effects: worker's occupation and birth cohort as well as plant's age, cohort, and industry.

B Model Details

B.1 Labor demand

Consider a problem of obtaining L units of composite labor at the minimal cost possible. Formally, consider a solution to the following problem

$$\min_{e,s} -w_l l - w_e e - w_s s,$$

s. t.

$$L = \left[A_l l^{\frac{\theta-1}{\theta}} + \bar{A}_e A_e(z,k) e^{\frac{\theta-1}{\theta}} + \bar{A}_s A_s(z,k) s^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}}$$

Input allocation satisfies

$$\frac{x}{y} = \left(\frac{\bar{A}_x A_x(z,k)}{A_y(z,k)} \right)^{\theta} \left(\frac{w_x}{w_y} \right)^{-\theta}$$

and hence

$$s = a_s(z,k)l, \qquad e = a_e(z,k)l, \tag{40}$$

where $a_x(z,k) = \left(\frac{\bar{A}_x A_x(z,k)}{A_l(z,k)} \right)^{\theta} \left(\frac{w_x}{w_l} \right)^{-\theta}$ It follows that

$$l = L \left[A_l + \bar{A}_e A_e(z,k) a_e^{\frac{\theta-1}{\theta}} + \bar{A}_s A_s(z,k) a_s^{\frac{\theta-1}{\theta}} \right]^{-\frac{\theta}{\theta-1}}$$

Normalize $A_l \equiv 1$. Then $a_x(z,k) = \left(\bar{A}_x A_x(z,k) \right)^{\theta} \left(\frac{w_x}{w_l} \right)^{-\theta}$, $\bar{A}_x A_x(z,k) a_x(z,k)^{\frac{\theta-1}{\theta}} = \bar{A}_x A_x(z,k)^{\theta} \left(\frac{w_x}{w_l} \right)^{1-\theta}$ and

$$l = L \left[1 + \bar{A}_e A_e(z,k)^{\theta} \left(\frac{w_e}{w_l} \right)^{1-\theta} + \bar{A}_s A_s(z,k)^{\theta} \left(\frac{w_s}{w_l} \right)^{1-\theta} \right]^{-\frac{\theta}{\theta-1}}$$

The cost of hiring one unit of composite labor is

$$W(z,k) = \left[1 + \bar{A}_e A_e(z,k)^{\theta} \left(\frac{w_e}{w_l} \right)^{1-\theta} + \bar{A}_s A_s(z,k)^{\theta} \left(\frac{w_s}{w_l} \right)^{1-\theta} \right]^{-\frac{\theta}{\theta-1}} (w_l + a_e(z,k) w_e + a_s(z,k) w_s)$$

$$= w_l \left[1 + \bar{A}_e A_e(z,k)^{\theta} \omega_e^{1-\theta} + \bar{A}_s A_s(z,k)^{\theta} \omega_s^{1-\theta} \right]^{-\frac{1}{\theta-1}}$$

B.2 Solving for productivity schedules

Since each worker supplies one efficiency unit of labor, the total size is

$$n = l \left(1 + a_e(z,k) + a_s(z,k) \right),$$

where $a_x(z,k) = \left(\frac{\bar{A}_x \bar{A}_x A_x(z,k)}{A_l(z,k)}\right)^\theta \left(\frac{w_x}{w_l}\right)^{-\theta}$. Moreover, (40) implies that

$$\frac{s}{n} = \left[1 + \left(\frac{\bar{A}_e A_e(z,k)}{\bar{A}_s A_s(z,k)} \frac{w_s}{w_e}\right)^\theta + \bar{A}_s A_s(z,k)^{-\theta} \omega_s^\theta\right]^{-1} \tag{41}$$

and

$$\frac{e}{n} = \left[1 + \left(\frac{\bar{A}_e A_e(z,k)}{\bar{A}_s A_s(z,k)} \frac{w_s}{w_e}\right)^{-\theta} + \bar{A}_e A_e(z,k)^{-\theta} \omega_e^\theta\right]^{-1} \tag{42}$$

Let $\hat{e}(n(z,k))$, $\hat{s}(n(z,k))$ mark the factor intensity in the data, where I explicitly indicated that the experience and skill shares in the data are function of plant size. We can re-write (41) and (42) as follows

$$\hat{s}(n(z,k))^{-1} - 1 = XY^{-1}\left(\frac{\omega_s}{\omega_e}\right)^\theta + Y^{-1}\omega_s^\theta$$

$$\hat{e}(n(z,k))^{-1} - 1 = X^{-1}Y\left(\frac{\omega_s}{\omega_e}\right)^{-\theta} + X^{-1}\omega_e^\theta$$

where $X = \hat{A}_e(z,k)^\theta$ and $Y = \hat{A}_s(z,k)^\theta$. The above system of equations yields the estimates of productivity schedules $\hat{A}_e$, $\hat{A}_s$. The solution is

$$X = \hat{A}_e(z,k)^\theta = \frac{\hat{s}(n(z,k))^{-1}\omega_e^\theta}{\hat{e}(n(z,k))^{-1}\hat{s}(n(z,k))^{-1} - \hat{e}(n(z,k))^{-1} - \hat{s}(n(z,k))^{-1}}$$

$$Y = \hat{A}_s(z,k)^\theta = \frac{\hat{e}(n(z,k))^{-1}\omega_s^\theta}{\hat{e}(n(z,k))^{-1}\hat{s}(n(z,k))^{-1} - \hat{e}(n(z,k))^{-1} - \hat{s}(z,k)^{-1}}$$

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