Federal Unemployment Reinsurance and Local Labor-Market Policies

Marek Ignaszak, Philip Jung, Keith Kuester*

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Abstract

Consider a union of atomistic member states. Idiosyncratic business-cycle shocks cause persistent differences in unemployment. Private cross-border risk-sharing is limited. A federal unemployment-based reinsurance scheme can provide transfers to member states in recession, which helps stabilize local unemployment. Limits to federal generosity arise because member states control local labor-market policies. Calibrating the economy to a stylized European Monetary Union, we find that moral hazard puts notable constraints on the effectiveness of federal reinsurance. This is so even if payouts are indexed to member state's usual unemployment rate or if the federal level pays only in severe-enough recessions.

Keywords: Unemployment reinsurance, fiscal risk sharing, labor-market policy, fiscal federalism, search and matching

JEL-Codes: E32, E24, E62, H77

*Ignaszak: Goethe University Frankfurt, Theodor-W.-Adorno-Platz 3, 60323 Frankfurt, Germany, ignaszak@uni-frankfurt.de. Jung: Technical University of Dortmund, Faculty of Business, Economics and Social Sciences, 44221 Dortmund, Germany, Philip.Jung@tu-dortmund.de; Kuester (corresponding author): University of Bonn, Adenauerallee 24-42, 53113 Bonn, Germany, keith.kuester@uni-bonn.de, and CEPR. Ignaszak acknowledges funding from the DAAD. Both Ignaszak and Kuester acknowledge funding by the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) under Germany's Excellence Strategy – EXC 2126/1– 390838866. Jung acknowledges support by the DFG under the title of Collaborative Research Centre 823. Kuester: Support by the Deutsche Forschungsgemeinschaft (DFG) through CRC TR 224 (Project C05) is gratefully acknowledged. Acknowledgements: We would like to thank Pierre Cahuc, Michael Krause, Dirk Krueger, and Thepthida Sopraseuth for very valuable comments. We would also like to thank participants at the Bundesbank-ADEMU workshop "Fiscal Policy in EMU," the workshop of the Economic and Employment Stability Network at U Nuremberg; and seminar participants at Bonn, Tübingen, Autonoma Barcelona, Bayreuth, Berlin, Oxford, and Warsaw.

1 Introduction

Countries that enter an economic (or monetary union) typically forfeit control over some policy instruments that they could otherwise use to stabilize economic activity. To make up for this, such economic unions usually come with a federal fiscal capacity, namely, a capacity that helps smooth the impact of region-specific shocks on consumption and economic activity, Feyrer and Sacerdote (2013). The question is how to implement such a federal capacity in the face of member states' incentives to free-ride; see Persson and Tabellini (1996). One way that ensures accountability, of course, is to allow for a permanent threat of abandoning the insurance mechanism, Phelan and Townsend (1991). The other way is to implement a fiscal transfer mechanism that is known to provide insurance *ex ante*, under a fixed set of rules, and designed with elements of accountability in mind.

The current paper looks at one such transfer mechanism: a federal unemployment-based reinsurance (RI) scheme. We have in mind the member state of a federal union. A federal fiscal capacity contributes toward the member state's budget whenever unemployment is high. Such federal payments provide consumption insurance. In addition, they help prevent an adverse fiscal loop by which a recession forces the government to raise taxes, which prolongs the recession further. The limits to the scope of the RI scheme arise because member states retain control over their local labor-market policies. Akin to the setting currently implemented in the US, the federal scheme seeks to ensure accountability. It does so either by providing transfers only when unemployment exceeds the member-states' usual level of unemployment or by providing transfers only if unemployment rises sufficiently much. The paper provides both a quantitative assessment for the European Monetary Union and pencil-and-paper intuition. The main finding is that the transition phase after federal RI is introduced notably constrains the generosity and effectiveness of federal RI.

We model a union of atomistic and *ex-ante* identical member states. All policy choices are made *ex-ante*, before heterogeneity is realized. In the member state, a need for unemployment benefits arises because risk-averse workers cannot self-insure against unemployment. An information asymmetry between workers and the member-state government means that the government will not provide full insurance. A need for other labor-market policies arises because the unemployment benefits are distortionary. At the member-state level, the modeling follows Jung and Kuester (2015): Each member state is subject to idiosyncratic business-cycle risk. Search and matching frictions amid wage rigidity give rise to inefficient and potentially persistent fluctuations in local unemployment. There are two roles for the federal fiscal capacity. One arises from market incompleteness: by assumption there is no private risk sharing between member-state households, nor can member-state governments borrow from each other. The other arises because member states keep their labor-market policy fixed over the business cycle, meaning that higher unemployment comes with higher taxes to balance the budget, which in turn perpetuates unemployment itself. Federal transfers prevent the rise in taxes and, thus, help smooth the business cycle. We calibrate this model to the countries of the European Monetary Union.

We first provide analytical intuition for the long run. The propositions that we derive highlight the importance of making sure that member states do not draw transfers from the federal scheme in normal times. In the quantitative experiments, therefore, we look at federal unemployment-based RI schemes in which member states cannot permanently free-ride on the federal level. Members with higher-than-usual unemployment (a positive "unemployment gap") receive transfers. Members with lower-than-usual unemployment provide more of the funding. The federal level decides about the generosity of transfers and the time horizon that it uses to determine the level of what is the "usual" level of unemployment in the member state.

As a starting point, we suppose that member states cannot change their labor-market policy when the federal scheme is implemented. This "fixed-policy" scenario shows the potential gains from federal RI. Under fixed policies, the optimal federal RI is designed such that it indexes payouts to a long-run average of local unemployment rates and that it replaces virtually all of the income that the member state loses to the recession. Besides, federal RI succeeds in stabilizing the local business cycle, reducing the standard deviation of local unemployment by four fifths. The welfare *gains* from federal unemployment-based RI amount to about 0.2 percent of life-time consumption.[1]

Against this background we, next, look at federal RI when member states can adjust their local labor-market policies. What we have in mind is a scenario in which the introduction of federal RI leads member states to reevaluate their labor-market policies once and for all. We look at two different scenarios: the "long view" scenario and the "transitional view" scenario. The "long view" supposes that member states focus their policy choice on the long-run level of welfare only, disregarding gains or losses during the transition phase after the federal scheme is introduced. If this is the case, we find that the optimal federal RI looks virtually identical to what it was in the "fixed-policy" scenario: federal RI is both generous and effective. Also, it remains optimal to compute the unemployment gap

[1] In the calibrated model, the costs of business cycles in autarky run to roughly 0.4 percent of life-time consumption. The reason for the comparatively large costs is that the calibration captures the large swings in unemployment that euro-area member states have seen.

relative to a long-run average of unemployment in the member state. This shows that making payouts contingent on the unemployment gap does successfully account for the incentives to free ride in the long run.

The "transitional view," instead, not only accounts for the long run but also for the period shortly after the federal RI scheme is introduced. We find that this consideration of the transition phase leads to rather different implications for federal RI. Under the "transitional view," the optimal federal payouts are less generous and they are shorter-lived. Still, payouts are not negligible. For a scheme, for example, in which the payouts are linear in the unemployment gap, we find that the optimal federal RI on average replaces between a little over 3 percent and a little over 4.5 percent of the income lost to a recession, depending on what policies the member states can adjust. This needs to be compared to full replacement absent the member states' response.[2] Importantly, though, in these optimal federal RI schemes, the definition of how to measure the "usual" level of unemployment also changes markedly. It is now optimal to measure the unemployment gap in such a way that it compares current unemployment to a relatively recent history of unemployment in the member state, namely, to the average unemployment that the member state has experienced over just the previous 1.5 years. Since unemployment itself can be more persistent than this, the payouts may no longer cover the entire period of high unemployment. Not only does the federal RI system, thus, fail to provide generous consumption insurance but it also fails to insulate the member-state budget from the persistent rise in costs that is due to higher unemployment. What this means is that the member state has to raise taxes in a recession, propagating the recession itself. All this combined, under the "transitional view" the stabilization gains that federal RI brings are notably smaller than under the "fixed-policy" and "long view" scenarios. Accounting for the transition phase and the response of member states, optimal federal RI, therefore, delivers smaller welfare gains (welfare gains now run to roughly 0.01 percent of life-time consumption only).

Why are large welfare gains from federal RI elusive even if all the schemes appropriately prevent permanent free-riding? The reason is simple: when member states change their labor-market policies upon the introduction of the federal scheme, unemployment can build up faster than the measure of the "usual" unemployment level in the member state. During the transition phase, therefore, the indexation of payouts to the unemployment gap need not suffice to fully deter free-riding. In turn, the shorter the length of the reference period

[2] Expressed differently, when unemployment rises, on average the federal level covers between 6 percent and 8.5 percent of the increase in outlays that the member state incurs for running the local unemployment insurance system.

is over which the "usual" unemployment rate is computed, the smaller the incentives to free-ride during the transition phase.

In addition, we also look at a–computationally more demanding–scheme that approximates the federal/state funding structure of the US unemployment insurance system where only in deep-enough recessions the federal level contributes to the costs of unemployment insurance in the member state. The scheme that we look at features a cutoff so that payouts from the federal level are made only when the unemployment gap is sufficiently large (in our case, when unemployment rises by 1.5 percentage points, or about 15 percent above its "usual" level). This federal RI scheme on average replaces about 10 percent of the income lost in a *severe* recession. Still, the stabilization gains are small. All told, the federal RI scheme with a threshold realizes a consumption-equivalent welfare gain of 0.016 percent of lifetime consumption. This gain is slightly larger than the one with a linear federal RI scheme, it still accounts for only about 7 percent of the gains that could be possible if member state's incentives were under control (such as in the fixed-policy scenario reported above).

In interpreting these results, it is important to stress what we show and what we do not show. We choose a simple federal unemployment-based reinsurance scheme because such schemes arguably have the political appeal of being easier to implement. We show that such a simple federal transfer scheme implies sizable transfers in an environment in which member-state-specific shocks mean that unemployment across member states differs persistently. Still, these transfers are neither large enough nor persistent enough to achieve the full welfare benefits that federal RI could achieve absent the member states' response. This is so even if the scheme has elements of long-run accountability. There are two reasons for this. On the one hand, the member states' incentives to free-ride in the near term render the optimal payouts from the scheme insufficiently persistent to provide the lasting fiscal buffer that is needed to provide stabilization gains. On the other, when they are made, the payouts are less generous than absent a member state's response. This notwithstanding, more complicated schemes could, of course, come with larger welfare benefits. Under commitment, for example, schemes that can condition on the source of cyclical fluctuations directly or on member states' labor-market policy choices could improve welfare relative to the simple scheme (recall the "fixed-policy" scenario in our paper). They may raise the question of implementability, however. Similarly, self-enforcing transfer mechanisms that have loan-like features might help support a federal fiscal capacity even under limited commitment; e.g. Ábrahám et al. (2022). To us, it remains a question for future research to see if such self-enforcing mechanisms could do so in an environment, like ours, where member states retain control over a a wide range of local policies.

The rest of the paper proceeds as follows. Next, we put the paper into the context of the literature. Section 2 spells out the model and the member states' and federal government's problems. The same section explains the transmission channel through which federal RI can provide stabilization and welfare gains in the current setting. Section 3 discusses the transmission channel of federal transfers. Besides, it shows analytically that federal unemployment-based RI distorts the entire labor-market policy mix unless the payouts are adequately indexed to the member state's unemployment experience. Looking at schemes that feature such indexation, Section 4 provides a numerical assessment that delivers the results that we mentioned above. A final section concludes.

Related literature

The current paper's insights are linked to an extensive literature on fiscal federalism; see Alesina and Wacziarg (1999) or Oates (1999) for references to the literature. A central reference for us is Persson and Tabellini (1996), who ask if fiscal risk sharing can induce local governments to underinvest in programs that alleviate local risk. Theirs is a static setting with two countries and two states of nature per country. One way to think about our paper is as an extension of the literature to a dynamic environment. Dynamics allow us to calibrate the potential quantitative gains from risk sharing and to separate considerations for the short and the long run, which we show to be important.

In our paper, there are two roles for the federal fiscal capacity. One arises from market incompleteness: by assumption there is no private risk sharing between member-state households, nor can member-state governments borrow from each other. The other arises from a restriction on local policies. Because local labor-market policy remains fixed over the business cycle, member states cannot smooth unemployment. Instead, higher unemployment comes with higher taxes to balance the budget, which in turn perpetuates unemployment itself.

Considerations for labor-market policies for single countries are discussed in Landais et al. (2018), Mitman and Rabinovich (2015), and Jung and Kuester (2015). Birinci and See (forthcoming) focus on economies with self-insurance by households, from which we abstract. We also do not allow for counter-cyclical local labor-market policies. Jung and Kuester (2015) assess these for a single country. Chodorow-Reich et al. (2018) and Hagedorn et al. (2013) discuss empirical considerations for benefits. Cahuc et al. (2018) look at countercyclical hiring subsidies. If we were to provide the member states access to such policies, this would further reduce the scope for federal unemployment-based RI; a scope which (due to the incentive effects on member states) we find to be limited to start with. A central component of our paper is that member states can adjust their labor-market policies in response to the federal scheme. Keeping member-state policies constant, instead, other papers find considerable scope for federal unemployment-based reinsurance. Examples are Cooper and Kempf (2004), who –like us– look at *ex-post* heterogeneity; or Ábrahám et al. (2019), Moyen et al. (2019), Enders and Vespermann (2021), or Dolls et al. (2018) who allow for *ex-ante* heterogeneity.

A motivation for the current paper is that empirical work shows that risk sharing is low in the euro area, see Furceri and Zdzienicka (2015) and Ferrari and Rogantini Picco (2023). Unemployment-based federal reinsurance provides a potential fiscal remedy for such a lack of risk sharing. Our paper contributes to the empirical evidence in two ways. First, we show that the member states' incentives to adjust their labor-market policy in a less employment-friendly way notably constrains the optimal extent of risk sharing. Second, we make an important conceptual contribution to the interpretation of risk-sharing regressions. In our setting, absent the member states' response, the transfers do not only provide consumption risk sharing but they also reduce aggregate risk. Namely, federal transfers help stabilize the business cycle. Thereby, they induce higher average employment.[3] At the same time, member states may seek to free-ride on the federal RI scheme. They do so by implementing policies that induce lower average employment. Both the employment-enhancing effect of federal RI due to stabilization and the employment-reducing of federal RI due to free-riding will not show up in empirical risk-sharing regressions. On net, we find that the incentives to free-ride limit all three: the stabilization gains, the employment gains, and the welfare gains that federal RI brings.

2 The model

There is a federal union that consists of a unit mass of atomistic, *ex-ante* identical member states. Member states are subject to member-state-specific productivity shocks. Member states are marked by subscript i. Member states control the local labor-market policies: unemployment insurance (UI) benefits, layoff taxes, and hiring subsidies. To finance the

[3]The link between greater employment stability and higher average employment in search models is discussed in detail in Hairault et al. (2010) and Jung and Kuester (2011), for example.

outlays, member states use a production tax that they level on domestic firms. They balance the budget period by period. There is no international borrowing and lending, nor is there self-insurance by households. A federal RI scheme makes unemployment-dependent transfers to the member-state government.[4] Time is discrete and runs from $t = 0$ to infinity.

2.1 The member-state economy

There are three types of agents in each member state: a unit mass of infinitely lived workers, an infinite mass of potential one-worker firms that produce a homogeneous final good, and the member-state government.

2.1.1 Workers

A worker's lifetime utility is given by

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta^t [\log(c_t^i) + \overline{h} \cdot \mathbb{I}(\text{not working}_t) - \iota \cdot \mathbb{I}(\text{search}_t)] \right\}. \tag{1}$$

E_0 denotes the expectation operator. $\beta \in (0, 1)$ is the time-discount factor. The worker draws utility from consumption, c_t. $\mathbb{I}$ is the indicator function. If not employed, the worker enjoys an additive utility of leisure $\overline{h}$ (or has a utility cost of unemployment if negative). Search for a job is a 0-1 decision. Workers are differentiated by a utility cost of search, ι, incurred only if the worker searches for a new job. Across both workers and time $\iota \overset{iid}{\sim} F_\iota(0, \sigma_\iota^2)$, where $F_\iota(\cdot, \cdot)$ marks the logistic distribution with mean 0 and variance $\sigma_\iota^2 = \pi \frac{\psi_\iota^2}{3}$. Here, $\psi_\iota > 0$ and π is the mathematical constant.

Workers own all firms in their member state in equal proportion. Ownership of firms is not traded. Workers cannot self-insure against income fluctuations through saving or borrowing. Letting Π_t^i mark the dividends that the member state's firms pay and w_t^i the wage that an employed worker earns, consumption of the worker is given by

$$\begin{aligned} c_{u,t}^i &:= b^i + \Pi_t^i \quad \text{if unemployed at the beginning of } t, \\ c_{e,t}^i &:= w_t^i + \Pi_t^i \quad \text{if employed at the beginning of } t. \end{aligned} \tag{2}$$

If the worker enters the period unemployed, the worker receives an amount b^i of unemployment benefits. The assumption is that the member state's government cannot observe the

[4] For the member state, the model and exposition here build extensively on Jung and Kuester (2015).

search effort of workers. The government, by assumption, conditions payment only on the worker's current employment status. A worker who enters the period employed receives wage income or, if separated, a severance payment from the firm equal to the period's wage.

Value of an employed worker

Let ξ_t^i be the separation rate of existing matches. Before separations occur, the value of an employed worker is

$$V_{e,t}^i = \quad \log(c_{e,t}^i) + [1 - \xi_t^i]\beta E_t V_{e,t+1}^i + \xi_t^i[V_{u,t}^i - \log(c_{u,t}^i)]. \tag{3}$$

The worker consumes $c_{e,t}^i$. With probability $1 - \xi_t^i$ the match does not separate and continues into $t + 1$. With probability ξ_t^i, instead, the match separates. The worker can immediately start searching for new employment. Therefore, the only difference of the newly-unemployed worker's value to the value of a worker, who was unemployed to start with, is that the separated worker receives the severance payment while the unemployed worker receives unemployment benefits (and, thus, lower consumption). $V_{u,t}^i$ is the value of a worker who starts the period unemployed.

Value of an unemployed worker and search

An unemployed worker decides to search for a job or not. Only those workers will decide to search whose disutility costs of search fall below a state-dependent cutoff value. We mark this cutoff by $\iota_t^{s,i}$. This cutoff is defined such that, at the cutoff, the utility cost of search just balances with the expected gain from search:

$$\iota_t^{s,i} = f_t^i \, \beta \, E_t \left[\Delta_{t+1}^i\right]. \tag{4}$$

Here $\Delta_t^i = V_{e,t}^i - V_{u,t}^i$ marks the gain from employment and f_t^i marks the job-finding rate. Using the properties of the logistic distribution, the share of unemployed workers who search is given by

$$s_t^i = Prob(\iota \leq \iota_t^{s,i}) = 1/[1 + \exp\{-\iota_t^{s,i}/\psi_s\}]. \tag{5}$$

With this, the value of an unemployed worker at the beginning of the period, before the search preference shock has been realized, is given by

$$\begin{aligned} V_{u,t}^i &= \log(c_{u,t}^i) + \overline{h} \\ &\quad - \int_{-\infty}^{\iota_t^{s,i}} \iota dF_\iota(\iota) + s_t^i \left[f_t^i \beta E_t V_{e,t+1}^i + [1 - f_t^i] \beta E_t V_{u,t+1}^i \right] \\ &\quad + (1 - s_t^i) \beta E_t V_{u,t+1}^i . \end{aligned} \tag{6}$$

In the current period, the worker consumes $c_{u,t}^i$ and enjoys utility of leisure $\overline{h}$ (first row). If the worker decides to search (second row), the utility cost is ι. The term with the integral is the expected utility cost of search. s_t^i is the *ex-ante* probability that the worker will search. With probability f_t^i the searching worker will find a job. In that case, the worker's value at the beginning of the next period will be $V_{e,t+1}^i$. With probability $(1 - f_t^i)$ the worker remains unemployed in the next period. If the worker does not search (third row), the worker remains unemployed.

2.1.2 Firms

Profits in the firm sector accrue to the workers, all of whom hold an equal amount of shares in the domestic firms. The decisions made by firms are dynamic and involve discounting future profits. We assume that firms discount the future using discount factor $Q_{t,t+s}^i$, where $Q_{t,t+s}^i := \beta \frac{\lambda_{t+s}^i}{\lambda_t^i}$, and where λ_t^i is the weighted marginal utility of the workers (the firms' owners):

$$\lambda_t^i := \left[e_t^i c_{e,t}^i + u_t^i c_{u,t}^i \right]^{-1} . \tag{7}$$

This reflects the fact that a mass e_t^i of workers are employed at the beginning of the period and a mass $u_t^i := 1 - e_t^i$ are unemployed.

Firms need a worker to produce output. A firm that enters the period matched to a worker can either produce or separate from the worker. Production entails a firm-specific resource cost, ϵ. This fixed cost is independently and identically distributed across firms and time with distribution function $F_\epsilon(\mu_\epsilon, \sigma_\epsilon^2)$. $F_\epsilon(\cdot, \cdot)$ is the logistic distribution with mean μ_ϵ and variance $\sigma_\epsilon^2 = \pi \frac{\psi_\epsilon^2}{3}$, with $\psi_\epsilon > 0$. One can show that the firm separates from the worker whenever the idiosyncratic cost shock, ϵ, is larger than a state-dependent threshold that we mark by $\epsilon_t^{\xi,i}$, and define later in equation (17). Using the properties of the logistic distribution, the separation rate can be expressed as

$$\xi_t^i = Prob(\epsilon \geq \epsilon_t^{\xi,i}) = 1/[1 + \exp\{(\epsilon_t^{\xi,i} - \mu_\epsilon)/\psi_\epsilon\}]. \tag{8}$$

Ex-ante, namely, before the idiosyncratic cost shock ϵ is realized, the value of a firm that has a worker is given by

$$\begin{aligned} J_t^i &= -\xi_t^i \left[\tau_\xi^i + w_t^i\right] \\ &\quad - \int_{-\infty}^{\epsilon_t^{\xi,i}} \epsilon \, dF_\epsilon(\epsilon) + (1-\xi_t^i) \, \left[\exp\{a_t^i\} - w_t^i - \tau_{J,t}^i + E_t Q_{t,t+1}^i J_{t+1}^i\right]. \end{aligned} \tag{9}$$

Upon separation, the firm is mandated to pay a layoff tax τ_ξ^i to the government and a severance payment of a period's wage w_t^i to the worker (first line). If the cost ϵ does not exceed the threshold, the firm will not separate. Rather, the firm will pay the resource cost, and the match will produce (second line). a_t^i is a member-state-specific labor-productivity shock. This is the source of the *ex-post* heterogeneity of member states. The firm produces $\exp\{a_t^i\}$ units of the good and pays the wage w_t^i to the worker. In addition, the firm pays a production tax $\tau_{J,t}^i$. A match that produces this period continues into the next.

The labor-productivity shock, a_t^i, evolves according to

$$a_t^i = \rho_a \, a_{t-1}^i + \varepsilon_{a,t}^i, \quad \rho_a \in [0,1), \; \varepsilon_{a,t}^i \sim N(0, \sigma_a^2).$$

A firm that does not have a worker can post a vacancy. If the firm finds a worker, the worker can start producing from the next period onward. Accounting for subsidies by the member state, the cost to the firm of posting a vacancy is $\kappa_v(1-\tau_v^i)$. $\kappa_v > 0$ marks a resource cost, and τ_v^i is the government's subsidy for hiring. In equilibrium, employment-services firms post vacancies until the after-tax cost of posting a vacancy equals the prospective gains from hiring:

$$\kappa_v(1-\tau_v^i) = q_t^i E_t \left[Q_{t,t+1}^i J_{t+1}^i\right], \tag{10}$$

where q_t^i is the probability of filling a vacancy.

Let v_t^i be the total mass of vacancies posted in the member state. Matches m_t^i evolve according to a constant-returns matching function:

$$m_t^i = \chi \cdot \left[v_t^i\right]^\gamma \cdot \left[[\xi_t^i e_t^i + u_t^i] s_t^i\right]^{1-\gamma}, \; \gamma \in (0,1). \tag{11}$$

Here, $\chi > 0$ is match efficiency. The mass of workers who potentially search is $\xi_t^i e_t^i + u_t^i$, with $\xi_t^i e_t^i$ being workers separated at the beginning of the period and u_t^i the mass of workers that enter the period unemployed. s_t^i is the share of those who do actually search. With this, employment evolves according to

$$e^i_{t+1} = [1 - \xi^i_t] \cdot e^i_t + m^i_t. \tag{12}$$

Total production of output is given by

$$y^i_t = e^i_t(1 - \xi^i_t)\exp\{a^i_t\}, \tag{13}$$

where $e^i_t(1-\xi^i_t)$ is the mass of existing matches that are not separated in t.
For subsequent use, define labor-market tightness as $\theta^i_t := v^i_t/([\xi^i_t e^i_t + 1 - e^i_t]s^i_t)$, the job-finding rate as $f^i_t := m^i_t/([\xi^i_t e^i_t + 1 - e^i_t]s^i_t) = \chi[\theta^i_t]^\gamma$, and the job-filling rate as $q^i_t := m^i_t/v^i_t = \chi[\theta^i_t]^{\gamma-1} = f^i_t/\theta^i_t$.

Dividends
Dividends in each member state arise from the profits generated by the firms that are located in the member state. Dividends are given by

$$\begin{aligned}\Pi^i_t \quad = \quad & -e^i_t\left[\int_{-\infty}^{\epsilon^{\xi,i}_t} \epsilon dF_\epsilon(\epsilon)\right] + e^i_t(1-\xi^i_t)\left[\exp\{a^i_t\} - w^i_t - \tau^i_{J,t}\right] - e^i_t\xi^i_t\left[w^i_t + \tau^i_\xi\right] \\ & - \kappa_v[1-\tau^i_v]v^i_t.\end{aligned} \tag{14}$$

2.1.3 Bargaining between firm and worker

At the beginning of the period, matched workers and firms observe the aggregate shock, a^i_t. Conditional on this, and *prior* to observing a match-specific cost shock ϵ_j, firms and workers bargain over the wage and the severance payment as well as over a state-contingent plan for separation. The firm will insure the risk-averse worker against the idiosyncratic risk associated with ϵ_j. The wage, w_t, thus, is independent of the realization of ϵ_j and the severance payment equals the wage. Anticipating this, the problem that firm and worker solve is

$$(w^i_t, \epsilon^{\xi,i}_t) = \arg\max_{w^i_t, \epsilon^{\xi,i}_t} (\Delta^i_t)^{1-\eta^i_t}(J^i_t)^{\eta^i_t}, \tag{15}$$

where η^i_t measures the bargaining power of the firm. We shall assume that η^i_t is linked to productivity according to $\eta^i_t = \eta \cdot \exp\{\gamma_w \cdot a^i_t\}$, with $\gamma_w \geq 0$. If $\gamma_w > 0$, the bargaining power of firms is low in recessions and high in booms. This introduces inefficient unemployment fluctuations. The first-order condition for the wage is as follows

$$(1-\eta^i_t)\, J^i_t = \eta^i_t\, \Delta^i_t\, c^i_{e,t}. \tag{16}$$

This states that after adjusting for the bargaining weights, the value of the firm equals the surplus of the worker from working, expressed in units of consumption when employed.
The first-order condition for the separation cutoff yields

$$\epsilon_t^{\xi,i} = \left[\exp\{a_t^i\} - \tau_{J,t}^i + \tau_\xi^i + E_t Q_{t,t+1}^i J_{t+1}^i\right] + \left[\beta E_t \Delta_{u,t+1}^i + \psi_s \log(1 - s_t^i) - \overline{h}\right] c_{e,t}^i. \quad (17)$$

2.1.4 The Federal RI scheme and market clearing

The current paper considers a specific form of the federal unemployment reinsurance scheme. Let $\mathbf{B}_F\left(u_t^i - u_t^{avg,i}\right)$ mark transfers from the federal level to the member-state government. These transfers are conditioned on the gap between member state's current unemployment, $u_t^i := 1 - e_t^i$, and a moving index of past unemployment,

$$u_t^{avg,i} := \delta u_{t-1}^{avg,i} + (1-\delta)u_{t-1}^i, \text{with } \delta \in [0,1). \quad (18)$$

Here δ controls the effective length of the reference period. For $\delta = 0$, the reference period would be last quarter's unemployment. For $\delta \to 1$, the reference point would the unconditional historical average.
All member states are subject to the same structure of the federal RI scheme. Let $\boldsymbol{\tau}_F$ mark a flat, time-independent contribution to the federal RI scheme, paid by each member state. Anticipating that member-state governments do not have access to international borrowing or lending (see Section 2.2.2), goods market clearing in each member state requires that in each of them

$$y_t^i + \mathbf{B}_F\left(u_t^i - u_t^{avg,i}\right) - \boldsymbol{\tau}_F = e_t^i c_{e,t}^i + u_t^i c_{u,t}^i + e_t^i \int_{-\infty}^{\epsilon_t^{\xi,i}} \epsilon \, dF_\epsilon(\epsilon) + \kappa_v v_t^i. \quad (19)$$

The left-hand side has the goods produced in the member state plus the net transfers received under the federal RI scheme. In equilibrium, goods are used for consumption (the first two terms on the right-hand side), for production costs, or for vacancy-posting costs. Once markets clear in all the member states, they also clear for the union as a whole.
In terms of accounting, in the calibration later on, we view the resources spent on retaining the match as intermediate goods, such that the definition of GDP is

$$gdp_t^i = y_t^i - e_t^i \int_{-\infty}^{\epsilon_t^{\xi,i}} \epsilon \, dF_\epsilon(\epsilon). \quad (20)$$

Market clearing expressed in units of GDP then is

$$gdp_t^i + \mathbf{B}_F\left(u_t^i - u_t^{avg,i}\right) - \boldsymbol{\tau}_F = e_t^i c_{e,t}^i + u_t^i c_{u,t}^i + \kappa_v v_t^i. \tag{21}$$

2.2 Government sector

There are two levels of government: the federal level and the local, member-state level. At the beginning of period $t = 0$, before any idiosyncratic shocks to member states have materialized, the federal government can set up a federal RI scheme, knowing the initial distribution of member states in the state space, and anticipating the response by member states and households. Period $t = 0$ is the first period in which the scheme will make payouts and collect contributions. The federal government is a first-mover. The federal RI scheme is implemented in a permanent manner and under full commitment. This choice of timing seems a reasonable first pass for many countries; we believe it is even more reasonable for the European Monetary Union/European Union, where changes to binding agreements often require unanimity. Immediately after the federal RI scheme is announced, member-state governments choose their permanent labor-market policies, taking the federal RI scheme as given. We describe each level of government in turn.

2.2.1 The federal government's problem

The federal RI scheme is characterized by the shape of payouts and the extent to which these payouts are indexed to the past unemployment experience in the member state. These, respectively, determine the short-term generosity of federal RI and the length of the reference period for unemployment. Throughout this paper, we parameterize the payout scheme. The federal RI scheme takes the form

$$\mathbf{B}_F(u_t^i - u_t^{avg,i}) = \alpha \cdot (u_t^i - u_t^{avg,i})\mathbb{I}(|u_t^i - u_t^{avg,i}| \geq \Phi), \ \alpha \geq 0, \Phi \geq 0. \tag{22}$$

Here α marks the generosity of federal RI conditional on making payouts (respectively, the intensity of taxation of the member state if the member state is in a boom). Φ marks a cutoff that makes sure that payouts are made in sufficiently deep recessions only (respectively, contributions are asked for in sufficiently large booms only). Let μ_t mark the distribution of member states across the possible states of the economy, in period t. Let $\tilde{\mu}_t$ be the induced distribution over the payout-relevant characteristics $(u_t^i, u_t^{avg,i})$. Throughout, we condition our solutions on the initial distribution of member states prior to period $t = 0$, assuming that at this time, there is no heterogeneity (so that all member states start from the same initial condition). Note that, once one does this, by the law of large numbers for any t both μ_t and $\tilde{\mu}_t$, are measurable already at the beginning of period 0.[5]

The federal government has access to international borrowing and lending at a fixed gross interest rate $1 + r = 1/\beta$. The federal RI scheme has to be self-financing in the sense that payouts or any debt must be financed completely by the federal RI taxes, so that

$$\sum_{t=0}^{\infty}(1+r)^{-t}\int\left(\mathbf{B}_F(u_t^i - u_t^{avg,i}) - \boldsymbol{\tau}_F\right)\, d\tilde{\mu}_t = 0. \tag{23}$$

Here, we integrate over the distribution of $(u_t^i, u_t^{avg,i})$ in all member states in the respective period t. Weighting all households in a member state equally, using (1) and the logistic distribution, after shocks have realized in period t a member state's utilitarian welfare function can be written as (see Jung and Kuester (2015) for a derivation)

$$W_t^i := E_t\sum_{k=t}^{\infty}\beta^k\left[e_k^i\log(c_{e,k}^i) + u_k^i\log(c_{u,k}^i) + \ (e_k^i\xi_k^i + u_k^i)(\Psi_s(s_k^i) + \overline{h})\right]. \tag{24}$$

The first term is the consumption-related utility of employed workers. The second term is the consumption-related utility of unemployed workers. The third term refers to the value of leisure and the utility costs of search.[6]

With this, the federal government's problem is to

$$\max_{\mathbf{B}_F(\cdot),\boldsymbol{\tau}_F} \quad \int_0^1\int W_0^i\, d\mu_0\, di \tag{25}$$

s.t. member states' policy response (see Section 2.2.2)
induced law of motion of member states' economies (earlier sections)
shape of the federal RI scheme as given by (18) and (22),
federal government's financing constraint (23).

Importantly, when choosing the shape of the federal RI scheme, the federal government

[5]The position of each member state i in the distribution is random. Since all risk is idiosyncratic, though, and member states are given equal weight in the federal planner's welfare function below, it does not matter which member state is in what position of the distribution.

[6]Here $\Psi_s(s_k^i) := -\psi_s\left[(1-s_k^i)\,\log(1-s_k^i) + s_k^i\log(s_k^i)\right]$. $\Psi_\xi(\xi_k^i)$, which is used further below, is defined in an analogous manner.

anticipates the response to the scheme by both the member states' governments and by the constituents of each member state.

2.2.2 The member-state government's problem

The member-state government takes the federal unemployment-based RI scheme as given. It then chooses state-and-time-independent labor-market policies (with production taxes balancing the budget). The member-state government's problem is given by

$$\max_{\{\tau_v^i, \tau_\xi^i, b^i, \tau_{J,t}^i\}} \int W_0^i \, d\mu_0 \tag{26}$$

$$\begin{aligned} \text{s.t.} \quad & \text{a given federal RI scheme } \mathbf{B}_F(\cdot), \boldsymbol{\tau}_F \\ & \text{law of motion for the member state's economy (earlier sections)} \\ & \text{the member-state government's budget constraint (27),} \end{aligned}$$

We model a one-time choice of labor-market instruments, with commitment to these values afterward.[7] $\tau_{J,t}^i$ then moves with the state of the business cycle, so as to clear the government's budget in each period. Being atomistic, the member-state government takes the federal RI scheme and decisions by other member states as given. It does anticipate, however, how its own choice of labor-market instruments affects its own local economy.[8] The member-state government faces the budget constraint

$$e_t^i(1 - \xi_t^i)\tau_{J,t}^i + e_t^i \xi_t^i \tau_\xi^i + \mathbf{B}_F\left(u_t^i - u_t^{avg,i}\right) = u_t^i b^i + \kappa_v \tau_v^i v_t^i + \boldsymbol{\tau}_F. \tag{27}$$

The left-hand side shows the member-state government's revenue from the production and layoff taxes, and the transfers received under the federal RI scheme. The right-hand side shows expenditure for unemployment benefits paid by the member state, the hiring subsidies paid by the member state, as well as the member state's fixed contribution toward the federal RI scheme.

[7] The exposition of problem (26) assumes that the member state optimizes over all labor-market policy instruments. In some of the sensitivity analysis of Section 4.4, we will also look at the case when the member state can only optimize some of the instruments.

[8] Note that the objective functions in (25) and (26) are identical. What differs are the constraints. The objective functions in the federal and local government's problem are identical because both levels of government take decisions at $t = 0$. All member states face the same initial conditions.

3 Benefits and costs of federal RI transfers

The current section discusses the benefits and costs of the federal RI transfers. It, thus, lays the ground for the quantitative assessment of the trade-offs that will follow in the later Section 4. Section 3.1 discusses how federal RI shapes the business cycle and how it provides insurance to workers against recessionary shocks; for given labor-market policy in the member state. Section 3.2 discusses how a federal unemployment-based RI system affects the member states' choice of policy in the first place.

3.1 The transmission channel of federal RI

Federal RI in the current paper has two benefits. First, the transfers provide insurance against the business-cycle shocks that affect the individual member state (the member state-specific shocks a_t^i). Second, federal RI helps smooth the business cycle itself by mitigating inefficient fluctuations in employment.

Federal RI transfers, $\mathbf{B}_F(u_t^i - u_t^{avg,i})$, mean additional resources to the member-state economy, equation (21). Thus, federal RI provides the aggregate economy with insurance against productivity risk. This insurance reaches the individual worker as follows. The federal level pays transfers to the member-state government. This loosens the member state's government budget constraint, equation (27). With a balanced budget rule and fixed local policies, the tax on production, $\tau_{J,t}^i$, falls. Dividends rise; compare equation (14). Since all workers in the member state share equally in the dividends, the federal RI payments are passed through to workers in a lump-sum fashion; compare equation (2).

The second effect of federal RI transfers is that they help stabilize employment in recessions. Wage rigidity means that wages tend to be inefficiently high in recessions, so that unemployment rises inefficiently much. Federal RI transfers come with lower taxes on production, as discussed earlier. If the fall in production taxes is persistent, the fall will raise the value of a (prospective) worker to a firm; compare equation (9). Therefore, federal RI transfers stimulate hiring and reduce the number of separations. Federal RI transfers, thereby, induce higher employment and, if provided in recessions, help stabilize the local business cycle. That is, federal RI reduces the underlying business-cycle risk itself.

3.2 Federal RI's effect on labor-market policy

If the federal government provides transfers in recessions, it provides consumption insurance and helps to smooth the member states' business cycles. At the same time, by conditioning on local unemployment the same federal transfers invite moral hazard by member states, namely, they give rise to the temptation to raise local unemployment itself. The current section provides analytical intuition for how federal transfers affect the member states' policy choices. This gives an indication, as well, of the sign of the response of local unemployment.

Here we focus on the steady state.[9] We seek to explore how federal RI affects member states' policy choices and employment when the federal scheme does not appropriately account for the "usual" level of local unemployment. To illustrate this, we consider a federal RI scheme in which u^{avg} is replaced by a given reference level $\overline{u}$. This level may or may not be equal to the long-run unemployment rate in the member state. This gives the following proposition.

Proposition. *Consider the economy described in Section 2. Let the member states solve problem* (26) *for a given federal RI scheme. Let the federal RI scheme be characterized by* $\mathbf{B}_F(u-\overline{u})$ *and* $\boldsymbol{\tau}_F$*, and let* $\mathbf{B}_F$ *be differentiable near the steady state. Let* $\mathbf{B}'_F(u-\overline{u})$ *mark the first derivative of the federal transfer function with respect to local unemployment. Let* $\Omega := \frac{\eta}{\gamma}\frac{1-\gamma}{1-\eta}$ *be the Hosios measure of search externalities and* $\zeta = \frac{\psi_s}{f(1-s)}\frac{1-e}{[\xi e+(1-e)]}\left[c_e - c_u\right]$ *be a measure of the tension between moral hazard and insurance of the* unemployed *in* each member state. *Focus on the steady state. The following labor-market policies and taxes implement the allocation that arises under optimal policy by the member state:*

$$\begin{aligned} b = \quad & \frac{(1-\beta)}{\beta}\tau_v\kappa_v\frac{\theta}{f}e + \zeta e \cdot \frac{[1-\beta(1-sf)(1-\xi)]}{\beta} \\ & + [\mathbf{B}_F(u-\overline{u}) - \boldsymbol{\tau}_F] \;+\; e\cdot\mathbf{B}'_F(u-\overline{u}), \end{aligned} \tag{28}$$

$$\tau_v = \quad [1-\Omega] + \frac{\eta}{1-\eta}\frac{\zeta}{\kappa_v\frac{\theta}{f}}, \tag{29}$$

$$\tau_\xi = \quad \tau_J + \tau_v\kappa_v\frac{\theta}{f} + \zeta(1-sf), \tag{30}$$

$$\tau_J = \quad \frac{1-e}{e}[b-\zeta sf] \;-\; \frac{\mathbf{B}_F(u-\overline{u}) - \boldsymbol{\tau}_F}{e}. \tag{31}$$

Proof. The proof is in Appendix A. □

Two dimensions of the federal RI scheme figure prominently. $\mathbf{B}_F(u-\overline{u}) - \boldsymbol{\tau}_F$ marks the steady-state net transfers. These are *inframarginal.* The proposition suggests that, all else

[9] Appendix A has the same derivations with cyclical fluctuations. The model's steady state is symmetric across the member states of the union. We therefore suppress superscript i whenever it is not strictly necessary.

equal, the higher the net transfers, the more fiscal space there is for the member state and the higher will the member state set unemployment benefits, equation (28). At the same time, the larger the net transfers, the lower will be taxes on firms τ_J, equation (31). It is precisely the latter effect that, over the business cycle and for fixed labor-market policies, allows federal transfers to stabilize employment.

$\mathbf{B}'_F(u-\overline{u})$, instead, marks federal RI transfers at the margin. The following corollary zooms in on these marginal effects.

Corollary. *Consider the same conditions as in the proposition above. Define the average duration of an unemployment spell as* $D \equiv \frac{1}{sf}$, *and the average time that an unemployed worker receives unemployment benefits as* $D_2 \equiv D - 1$.[10] *Define the elasticity of duration* D_2 *with respect to a sequence of unanticipated increases of unemployment benefits as* $\epsilon_{D_2,b} = \frac{D}{D_2}\frac{f(1-s)}{\psi_s}$. *Assume that* $\beta \to 1$ *and that the Hosios condition* ($\Omega = 1$) *holds. Further assume that inframarginal net transfers are zero, that is,* $\mathbf{B}_F(u-\overline{u}) = \boldsymbol{\tau}_F$.
Then, the following characterizes the local policies that the member-state government sets

$$
\begin{aligned}
b &= \frac{1}{1+D\epsilon_{D_2,b}}\,w \;+\; \frac{D\epsilon_{D_2,b}}{1+D\epsilon_{D_2,b}}\,[\, e\mathbf{B}'_F(u-\overline{u})\,] && (32)\\
\kappa_v\frac{\theta}{f}\tau_v &= \frac{\eta}{1-\eta}\,D\left(b-[\, e\mathbf{B}'_F(u-\overline{u})\,]\right) && (33)\\
\tau_\xi &= \tau_J + \kappa_v\frac{\theta}{f}\tau_v + D_2\left(b-[\, e\mathbf{B}'_F(u-\overline{u})\,]\right) && (34)\\
\tau_J &= [(1-e)\,\mathbf{B}'_F(u-\overline{u})]. && (35)
\end{aligned}
$$

Note also that, in equilibrium, dividends are zero, so that $b = c_u$ *and* $w = c_e$.

Proof. This is a special case of the proposition. See Appendix A for the proof. □

In each of the equations of the corollary, the terms in square brackets refer to the effects that marginal payouts of the federal RI scheme have on the local policies. Absent federal RI, the member state sets the labor-market policy mix in a way that is familiar from Jung and Kuester (2015). Namely, unemployment benefits reflect the direct resource costs of benefits and their effect on the unemployed worker's search effort during the entire unemployment spell. Hiring subsidies in turn reflect that an unemployed worker who is hired saves the government unemployment benefits. Layoff taxes reflect the fiscal externalities of a layoff. Namely, a match that separates no longer pays production taxes. In addition, for bringing the newly-unemployed worker back into employment the government pays hiring subsidies. Next, during the unemployment spell, the government expends unemployment benefits.

[10] In the current setting D and D_2 differ because the first period of joblessness is covered by severance payments from the firm. This is without material consequence.

Last, absent federal RI, the set of labor-market policies is self-financing, obviating the need for production taxes.
Federal unemployment-based RI distorts the labor-market policy mix toward a generally less employment-friendly outcome. To see this, focus on the term $e\mathbf{B}'_F(u, \overline{u})$. This is the product of the mass of workers that could still be moved into unemployment (that is, the mass of employed workers e) and the marginal payouts from the federal level, $\mathbf{B}'_F$. Focus first on equation (32). $D\epsilon_{D_2,b} > 0$ is the micro-elasticity of the average duration of an unemployment spell with respect to a permanent increase in unemployment benefits, see Appendix B. For a given micro-elasticity and wage w, more generous federal RI at the margin unambiguously leads the member state to reduce the gap between benefits and wages. In its own local moral-hazard-insurance trade-off between insuring workers and incentivizing search, the member state, therefore, opts for more insurance. Equation (33) gives the local effect of federal transfers on the hiring subsidy. On the one hand, benefits rise making the subsidy more important. On the other, unemployment no longer is as costly for the member state since the marginal unemployed worker generates federal transfers for the member state. For a given wage and micro elasticity, the difference in the round bracket is negative: Hiring subsidies fall when federal RI is more generous. The effect of the marginal federal payout on the size of layoff taxes, instead, is ambiguous: in (34) some of the outlays rise and others fall. What is important to note, however, is that if $\mathbf{B}'_F(u - \overline{u}) > 0$, the layoff taxes no longer are high enough to make firms internalize the entire fiscal costs that a layoff imposes. The way to see this is that, in this case, the provision of federal RI at the margin comes with positive production taxes in the steady state, see (35). These are needed to balance the member-state government's budget. The size of the rise in production taxes is such that the member-state government, so as to generate a marginal transfer of $\mathbf{B}'_F(u - \overline{u})$, incurs a fiscal short-fall of the same size for each unemployed worker ($u = 1 - e$).
In order to see some of the quantitative implications of the corollary, it may be useful to use equation (32) for a back-of-the-envelope calculation. Our calibration in Section 4.1 will target a micro-elasticity of the duration of unemployment with respect to benefits of $D\epsilon_{D_2,b} = 0.5$ in the steady state absent federal RI, a number that is in the mid-point of estimates in the literature, Schmieder and von Wachter (2016). The direct effect, that is, for a given elasticity and given wages, of a federal RI scheme on the replacement rate would then be $\frac{\partial b/w}{\partial B'_F/w} = \frac{D\epsilon_{D_2,b}}{1+D\epsilon_{D_2,b}} e = \frac{1}{3}e$. That is, with an employment rate of $e \approx 0.9$, the direct effect of federal RI on the replacement rate is sizable. If the federal level replaces

one percent of the wage of a worker, for example, the direct effect is that the replacement rate rises by 0.3 percentage points.

In light of the potential costs and benefits of federal RI discussed here, the next section turns to a quantitative assessment. From the corollary above, we know that any federal RI scheme that is not designed such that $\mathbf{B}'_F = 0$ will distort the member state's labor-market policy mix. In what follows, therefore, we will only look at federal RI schemes in which the member state cannot indefinitely extract resources from the federal level, that is, in which the incentives to free-ride are taken care of in the long run. Toward this end, we will look at such federal unemployment-based reinsurance schemes that index the payout of federal transfers to the gap between current unemployment and the "usual" level of unemployment in the member state, as in (22). This "usual" level of unemployment will eventually reflect the influence of the member state's policy choices on local unemployment. One of the results of the next section is that, regardless, federal RI remains distortionary. This is so because the incentives to free-ride that we discussed above remain in place during the transition period that starts immediately after federal RI is introduced. In this episode the measure of the "usual" level of unemployment catches up with a new reality only gradually. This explains why, later, we find that the transition period notably affects both the optimal generosity of federal RI (the slope $\mathbf{B}'_F$) and the period over which the member state's usual unemployment rate is best calculated (parameter δ of the federal RI scheme).

4 Optimal federal reinsurance

This section presents the paper's numerical results regarding the gains in stabilization and welfare that a well-designed unemployment-based RI scheme can achieve. Toward a numerical assessment, Section 4.1 describes the calibration approach. Section 4.2 describes the computation. The quantitative results regarding the optimal federal scheme are discussed thereafter. Section 4.3 discusses the design of linear federal RI schemes, that is, schemes that do not feature thresholds ($\Phi = 0$). Section 4.4 discusses sensitivity analysis, including allowing member states to adjust a limited set of instruments only. Section 4.5 discusses the gains associated with a threshold scheme ($\Phi > 0$).

4.1 Calibration

Our aim is to calibrate the model to a union of countries, each of which resembles a "generic" member of the European Monetary Union (EMU henceforth).

Second moments of the data. We obtain country-level time series for 14 euro-area member states for the sample period 1995Q1 to 2019Q4.[11] We first extract the cyclical component for each time series by applying a linear trend (or, if possible, simply demean).[12] For each country, we calculate moments. Last, we construct moments for a "generic" euro-area member state by computing population-weighted averages of the country-level moments. The business-cycle properties of the data are reported in Table 1. All the

Table 1: Business-cycle properties of the data

		gdp	*c*	*lprod*	*e*	*urate*	*w*
Stand. dev.		3.87	3.45	1.96	3.04	26.16	1.90
Autocorr.		0.96	0.96	0.93	0.99	0.99	0.92
Correlations	*y*	1.00	0.84	0.57	0.81	-0.61	0.29
	c	-	1.00	0.50	0.66	-0.50	0.27
	lprod	-	-	1.00	0.15	-0.14	0.36
	e	-	-	-	1.00	-0.74	0.28
	urate	-	-	-	-	1.00	-0.10

Notes: Summary statistics of the data (quarterly). Series are labeled like their counterparts in the model or as described in the text. All data are quarterly aggregates, in logs (including the unemployment rate), and multiplied by 100. We report the cyclical component after applying a linear trend. The exception is the log unemployment rate, which we demean only. Entries can be interpreted as percent deviation from the steady state. The first block reports the standard deviations and autocorrelations. The second block reports cross-correlations of time series within the typical country. The sample is 1995Q1 to 2019Q4. All entries are population-weighted averages of member-state-level moments.

moments refer to data measured at a quarterly frequency and in percentage deviations from trend.

The first block of the table reports the standard deviation and first-order autocorrelation; the second block reports the cross-correlation of the main aggregates at the country level. All series are from Eurostat and seasonally adjusted. Output y is real gross domestic product (chain-linked volumes). Consumption c is consumption by households and non-profits divided by the GDP deflator. Labor productivity, $lprod := \frac{gdp}{e(1-\xi)}$, is measured as GDP divided by employment (heads). The unemployment rate does not require transformation.[13] The wage is the ratio of wages and salaries per employee deflated by

[11]The countries are: Belgium, Germany, Ireland, Greece, Spain, France, Italy, Cyprus, Luxembourg, Malta, Netherlands, Austria, Portugal, and Finland.

[12]We take a linear trend such that the fall in GDP in several member states after 2008 and the commensurate rise in unemployment are left as part of the cyclical component of the time series.

[13]The model counterpart of the unemployment rate is $urate := (e\xi + u)s/[(e\xi + u)s + e(1-\xi)]$ (the mass

the GDP price index. The business cycle in the euro area is volatile and fluctuations are persistent; in particular those of unemployment. Unemployment also is countercyclical, meaning that it might serve as a good point of reference for federal transfers.

Model. Targets and parameters. Table 2 summarizes the calibration of the model's parameters. One period in the model is a month. Three of the model's parameters are directly linked to the business cycle: the standard deviation of the productivity shock, σ_a, the wage rigidity parameter, γ_w, and the dispersion of the match continuation costs, ψ_ϵ. We choose these so as to bring the model as close as possible to matching three business-cycle targets: the standard deviation of measured labor productivity, the standard deviation of the unemployment rate, and the relative standard deviation of the job-finding and separation rate. The first two targets' values are taken from Table 1. The third target is that the separation rate is 60 percent as volatile as the flow rate from unemployment to employment, in line with the shape of the cyclical fluctuations of European OECD countries' labor markets documented in Elsby et al. (2013). In searching for these parameters, we constrain the response of the wage to a positive productivity shock to be non-negative on impact. This limits the range of admissible values for γ_w, in particular, and means that the aforementioned targets, in practice, do not need to be hit exactly. Conditional on the remaining parameterization of the model (discussed below), the targets deliver $\sigma_a = 0.0039$, $\gamma_w = 12.95$,[14] and $\psi_\epsilon = 2.31$.

The other parameters are chosen based on targets for the steady state of the model or based on other outside evidence. The monthly discount factor, $\beta = 0.997$ implies that the households discount the future at 4 percent annually. In order to match an average unemployment rate (*urate*) of 9.5 percent, we set $\overline{h} = -0.047$. We set $\psi_s = 0.196$ such that the model matches micro-level evidence on search behavior. Namely, we target an elasticity of the average duration of unemployment with respect to an increase in unemployment benefits for the individual household of 0.5 (the underlying experiment is that of a permanent increase in benefits, holding everything but search behavior constant). This value of one half is reported in Schmieder and von Wachter (2016), who look at the median value of said elasticity across 18 studies. The micro-elasticity of unemployment duration is an important determinant of how much the member-state government adjusts the level of local unemployment benefits in response to federal RI transfers, recall the discussion

of non-employed workers who search divided by the labor force).

[14]The value of γ_w implies that for a 1 percent negative productivity shock, the bargaining power of firms falls by roughly 13 percent.

Table 2: Parameters for the baseline

	description	value	target
Preferences			
β	time–discount factor	0.997	putative real rate of 4% p.a.
$\bar{h}$	value of leisure.	-0.047	st.-st. u rate of 9.5 %
ψ_s	dispers. search cost	0.20	micro-elasticity, Schmieder and von Wachter (2016).
Vacancies and matching			
κ_v	vac. posting cost	1.12	EMU avg. monthly job-finding rate.
γ	match elasti. wrt v	0.30	Petrongolo and Pissarides (2001).
χ	match-efficiency	0.13	qtrly job fill rate 71%, den Haan et al. (2000).
Wages			
η	firms' st.-st. barg. p.	0.30	Hosios condition.
γ_w	cyclic. barg. power	12.95	unemployment volatility.
Production and layoffs			
μ_ϵ	mean idios. cost	0.45	continuation costs 1/3 of output.
ψ_ϵ	dispers. cost shock	2.31	rel. vola. job-f., sep. rate, Elsby et al. (2013).
ρ_a	AR(1) prod. shock	0.988	qtrly persistence of prod. shock of 0.96.
$\sigma_a \cdot 100$	std. dev.	0.39	standard deviation of measured *lprod*.

Notes: The table reports the calibrated parameter values in the baseline economy.

of the corollary's equation (32) in Section 3.2. Matching the micro elasticity, thus, is of first-order importance.

The vacancy posting cost of $\kappa_v = 1.12$ replicates the EMU-average monthly flow rate from unemployment to employment (sf) of 7.5 percent, compare Elsby et al. (2013). For reference, this gives an average cost per hire net of the hiring subsidy of roughly two monthly wages. We set the elasticity of the matching function with respect to vacancies to $\gamma = 0.3$. This is within the range of estimates deemed reasonable by Petrongolo and Pissarides (2001). The matching-efficiency parameter is set to $\chi = 0.127$ so as to match a quarterly job-filling rate of 71 percent. We take the latter target from den Haan et al. (2000). The bargaining power of firms in the steady state is set to $\eta = 0.3$ so as to meet the Hosios (1990) condition.

The average idiosyncratic cost of retaining a match is set to $\mu_\epsilon = 0.45$. This parameter governs the average costs of continuing a match. We set the parameter such that, in the steady state, the costs associated with continuing an employment relationship amount to 1/3 of output. Last, we set the serial correlation of the productivity shock to $\rho_a = 0.988$. This translates into a quarterly persistence of the productivity shock of 0.965, which is within the range of values entertained in the literature.

Implied business-cycle statistics of the model. Table 3 reports business-cycle statistics for the calibrated model. The moments reported here refer to quarterly averages of the monthly observations in the model. They, thus, are directly comparable to the moments in the data that we reported in Table 1. The calibration matches the rough

Table 3: Business-cycle properties of the model

		gdp	*c*	*lprod*	*e*	*urate*	*w*	*sf*	ξ
Standard dev.		4.85	3.75	1.89	3.08	26.16	1.74	18.42	13.40
Autocorr.		0.99	1.00	0.95	1.00	1.00	1.00	0.98	0.99
Correlations	*y*	1.00	0.99	0.92	0.98	-0.97	0.97	0.99	-1.00
	c	-	1.00	0.84	1.00	-1.00	1.00	0.95	-0.99
	lprod	-	-	1.00	0.82	-0.80	0.80	0.96	-0.91
	e	-	-	-	1.00	-1.00	1.00	0.94	-0.98
	urate	-	-	-	-	1.00	-1.00	-0.93	0.98
	w	-	-	-	-	-	1.00	0.93	-0.98
	sf	-	-	-	-	-	-	1.00	-0.99

Notes: Second moments in the model. All data are quarterly aggregates, in logs and multiplied by 100 in order to express them in percent deviation from the steady state. Note: the series for the unemployment rate is in logs as well. The first row reports the standard deviation, the next row the autocorrelation, followed by contemporaneous correlations. Based on a first-order approximation of the model.

outline of the euro-area member states' business cycles; in particular, a volatile and persistent unemployment rate amid strong fluctuations in consumption and GDP.

Implied steady state. Table 4 reports selected steady-state values for this baseline calibration, including the labor-market policies that the member state chooses. The optimal replacement rate in the steady state ($b/w = b/[c_e - \Pi]$) is 63 percent, a reasonable value for the euro area (compare Christoffel et al. 2009). The optimal vacancy subsidy is $\tau_v = 0.61$. This amounts to a subsidy per actual hire of roughly 3.5 monthly wages. The optimal layoff tax equals approximately 10 monthly wages, reasonable given the long average duration of unemployment spells in the EMU and the corresponding fiscal costs.

The macro-elasticity of unemployment with respect to local benefits. The proposition and corollary in Section 3.2 show how federal transfers affect the member-state government's choice of its local labor-market policy instruments. As discussed there, these

Table 4: Steady-state values

Labor-market policy			Output and consumption		
b	local UI benefits	0.37	gdp	GDP	0.60
τ_v	vacancy posting subsidy	0.61	c_e	consumption employed	0.59
τ_ξ	layoff tax	6.33	c_u	consumption unemployed	0.37
Labor market			Other variables		
ξ	separation rate	0.009	Π	dividends	0.004
f	job-finding rate	0.083	Δ	gain from employment	5.14
s	search intensity	0.90	J	value of employ.-serv. firm	1.30
e	employment	0.903	τ_J	tax on firms	0.002
u	unemployment	0.0907	D	duration of unempl. (mths)	13.3
$urate$	unemployment rate	0.095	$\epsilon_{D_2,b}^{(*)}$	elast. u-duration wrt. b	0.047

Notes: Selected steady-state values for the baseline economy. $^{(*)}$ the elasticity is computed as $\epsilon_{D_2,b} = \frac{D}{D_2}\frac{f(1-s)}{\psi_s}\beta$.

policy responses to federal RI give an indication of the *sign* of how local unemployment changes when federal RI is introduced. We cannot, however, derive what this implies for the full *size* of the response of local unemployment in closed form. The reason is that the effect of such policy on local unemployment (and, thus, welfare and the size of federal payouts) depends not only on the micro-elasticity of local unemployment with respect to a local policy change, but on the *macro-elasticity*. In the calibration, the *micro-elasticity* of unemployment with respect to a permanent increase in local benefits is 0.5 (as targeted). It is important to target this elasticity because it appears prominently in the corollary of Section 3.2. At the same time, this micro-elasticity by definition only accounts for how a change in benefits directly affects the income of the unemployed and, through this, their search behavior. The reason why the macro effects also matter is that in equilibrium a change in unemployment benefits also changes the wages of the employed as well as the firms' hiring and separation policies (because wages change and because benefits have to be financed). We cannot derive these macro effects in closed form, but we can deduce them numerically using the calibrated model toward this end. Keeping the other instruments (layoff taxes and hiring subsidies) fixed, in the calibrated baseline, that is, absent federal RI, the elasticity of unemployment with respect to a *permanent* change in benefits is 3.4, accounting for all equilibrium effects.[15] The empirical size of the macro-elasticity

[15] In our model, transitory increases in benefits would, of course, be associated with both smaller macro and micro elasticities of unemployment.

of unemployment is disputed; this is so even for temporary increases in unemployment benefits, see the difference in findings in Chodorow-Reich et al. (2018) and Hagedorn et al. (2016), for example. The elasticity here of 3.4 is within the range of estimates suggested by the literature. It aligns well, for example, with the empirical estimates for Sweden that Fredriksson and Söderström (2020) present. Their empirical work, like the elasticity reported here, focuses on the effect of permanent changes in benefits, while controlling for other policy dimensions.

Marginal federal generosity, labor-market policy, and the macro-economy. Next, we report how the generosity of *permanent* federal payouts quantitatively affects local policy choices and local unemployment in the non-stochastic steady state. The numbers that we provide in this respect account for the macro-economic feedback effects and they account for the fact that federal unemployment-based reinsurance induces the member states to adjust *all* of their policy instruments, not only unemployment benefits.[16] To be concrete, consider that starting from autarky, a linear federal RI scheme ($\Phi = 0$) is introduced that does not index payouts to the resulting new long-run level of unemployment ($\delta \to 1$). Suppose that the federal scheme is generous, featuring $\alpha = 0.06$. This value of α means that of the output that is lost when the average local worker becomes unemployed, the federal level replaces 10 percent (in autarky $gdp = 0.6$, compare Table 4). We adjust federal taxes $\boldsymbol{\tau}_F$ so as to make federal RI self-financing in the new steady state. Comparing the two non-stochastic steady states, this scheme induces the member states to increase local unemployment by a full 3 percentage points.The reason is simple: with the full set of instruments at its disposal, the member-state government in autarky is indifferent between having a marginal worker employed or unemployed, the information frictions provided. The member state, thus, lets unemployment adjust elastically to the federal transfer. In terms of instruments, the replacement rate b/w rises by 0.7 percentage points. The layoff tax falls by about 8.6 percent and hiring subsidies fall by 1.6 percentage points. That is, all of the labor-market instruments move in a less employment-friendly direction, just as the discussion in Section 3.2 suggested.

[16] Here we report the effect of federal RI on the non-stochastic steady state. The effect on policy choices and unemployment reported here, therefore, neglects the positive effects on employment that come from business-cycle stabilization. The numerical analysis below, starting with Section 4.3 accounts for both the business cycle effects and the steady state.

4.2 Numerical implementation of optimal policy exercises

Our paper starts from a setting in which federal RI can improve welfare, recall Section 3.1. We ask how much of these benefits remain when the design of the federal RI scheme has to account for the member states' policy responses, recall Section 3.2.

Ideally, we would wish to look for the optimal federal RI scheme simultaneously in three dimensions: the generosity upon making federal RI payouts (parameter α in (22)), the threshold for the unemployment gap beyond which payouts are made (parameter Φ in (22)), and the time it takes before the "usual" unemployment rate catches up with actual unemployment (governed by parameter δ in (18)). Doing so requires repeatedly solving the model globally, and finding best responses for three parameters and three policy choices (unemployment benefits, hiring subsidies, and layoff taxes). We did not find this joint approach computationally feasible. Instead, we proceed as follows.

Most of the results, that we will show, will refer to federal schemes that *do not* feature thresholds, that is, for which $\Phi = 0$. In this case, we can solve the model by perturbation methods, which are fast and which, thus, allow for the many solutions of the model (and evaluations of welfare) that we need. We allow for a continuous choice of the generosity α and a fine grid for parameter δ.[17] Since non-linearities are an important characteristic of the search and matching model and we seek to evaluate welfare accurately, we solve the model through a fourth-order perturbation. We apply pruning throughout. Technically, for solving the model, we rely on the routines by Levintal (2017).[18] Appendix C describes the algorithm in more detail. The results are shown in Section 4.3.

Thereafter, so as to evaluate the implications for the effectiveness of federal RI, we solve the game between the federal level and the member states using a global solution of the model. We do so for a given way of computing the usual unemployment rate (that is, for one fixed parameter δ) and for a given threshold Φ. In this context, we look for the optimal generosity of payouts α on a finer grid. To reduce complexity still further, we allow the member state to optimize only one of its policies, unemployment benefits, choosing values from a grid. Appendix E describes this algorithm. The results of this exercise with a threshold for payouts are shown in Section 4.5.

Throughout, unless noted otherwise, we use the calibrated non-stochastic steady state as

[17] We search for δ on a grid implying a half-life $\in [1/4, 1/2, 3/4, 1, 1.5, 2, 2.5, 3, 3.5, 4, 6, 8, 10, 100, 1000]$ years. So, for the largest value, the measure of the usual level of unemployment essentially is not affected by actual unemployment.

[18] For computing moments, we also extend the formulae for the first moments that Andreasen et al. (2018) provide for third-order approximations to fourth order.

the initial state of the economy on which the welfare evaluation is conditioned. That is, we assume that all member states enter period $t = 0$ (the first period) at the non-stochastic steady state implied by the calibration (recall that the baseline does not have any federal RI, compare Table 4 for the steady state). To be clear, with this, *ex-ante* all member states are identical. *Ex-post*, however, member states will be heterogeneous so that there is a role for the federal transfer scheme.

4.3 Optimal federal unemployment reinsurance (the linear case)

We now turn toward analyzing the shape and scope of an *ex-ante* optimal federal RI scheme for our calibrated model of the euro area, focusing on schemes that do not feature thresholds. We build the results in steps. To set the stage, section 4.3.1 illustrates the scope for federal RI when member states do not adjust their policies. Section 4.3.2 looks at optimal federal RI when member states can adjust their policies but do so disregarding the transition period. Section 4.3.3 then shows the scope for federal RI when accounting for both the transition period and member states' responses. We show that the gains are smaller and the optimal reference period for computing the "usual" level of unemployment much shorter than in the two earlier scenarios. Section 4.3.4 provides a discussion of how to interpret the results.

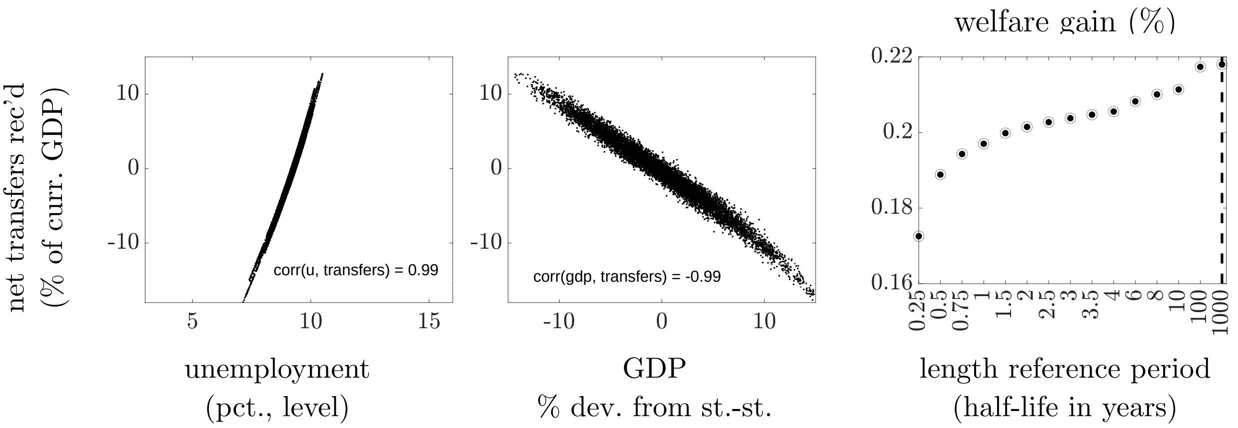
4.3.1 Optimal federal RI absent the member-states' response

To set the stage, we document the potential gains from federal unemployment-based reinsurance if the member state cannot adjust its labor-market policies in the current setting. That is, we hold member states' labor-market policies fixed. Figure 1 shows the results. For the optimal scheme, the left and center panel plot the net transfers from the federal RI scheme (y-axis, as a share of current GDP) against unemployment (x-axis, left panel) and GDP (x-axis, center panel), respectively. The right panel shows how the consumption-equivalent welfare gains from federal RI depend on the half-life of the reference level of unemployment, $u_t^{avg,i}$.

Absent the member states' response, the optimal federal RI scheme indexes to a long-run average of the unemployment rate (vertical dashed line in the right panel). This implies a virtually linear relation between payouts from the federal scheme and local unemployment (left panel).[19] The optimal scheme is generous, too: the implied transfers make up for most

[19]Absent a response by the member state, there is no need for a reference to the "usual" level of

Figure 1: Optimal federal RI with fixed local policies



Notes: Optimal federal RI when member states do not adjust their labor-market instruments. Left panel and center panel: Net transfers received $\mathbf{B}_F - \boldsymbol{\tau}_F$ (y-axis, as a share of current GDP) against unemployment, u, (x-axis, left panel) or percent deviation of GDP from its mean (x-axis, center panel). Based on simulations of 10,000 periods under the optimal federal RI scheme. Right panel: consumption-equivalent welfare gain (y-axis, in percent) associated with the optimized federal RI scheme as a function of the half-life of the reference period for unemployment (x-axis, half-life in years), the half-life being defined as $\frac{1}{12}\frac{\log(0.5)}{\log(\delta)}$. A vertical dashed line marks the largest gain.

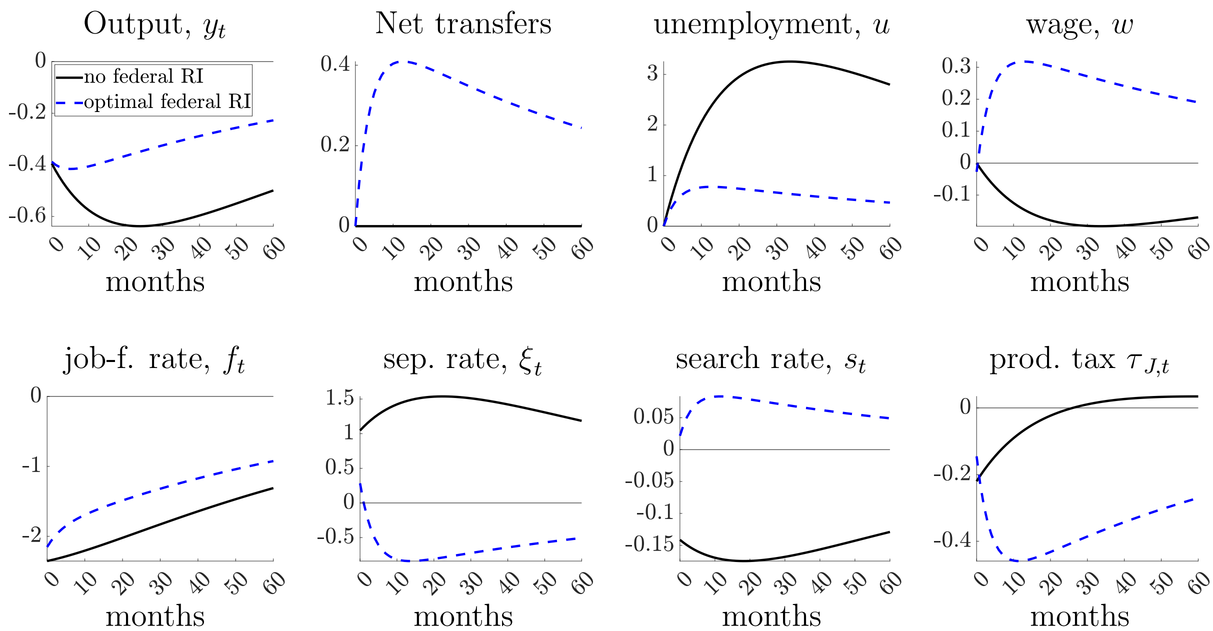
of the output lost in recessions (center panel); *vice versa*, member states that experience a boom make commensurate transfers. Importantly, the long reference period for the measure of what constitutes the usual level of unemployment means that federal payments are well-timed (the correlation of transfers with GDP is -0.99). In light of all this, the optimal federal RI scheme results in a notable consumption-equivalent welfare gain, namely, a gain that is equivalent to about 0.22 percent of life-time consumption (right panel).[20]

Figure 2 zooms in on the stabilization gains that the optimal federal RI scheme provides. The figure shows the impulse response of the economy to a recessionary productivity shock. Solid black lines refer to the case absent federal RI (the calibrated baseline economy). Wage rigidity means that the shock is propagated through the labor market. The job-finding rate falls, the separation rate rises, and workers search less. As a result, log unemployment rises by a little over 3 percent (the unemployment rate rises by about 0.4 percentage point). This means that output falls by more, and more persistently than the fall in labor productivity alone would suggest (top left panel). The response of the production tax (lower right panel)

unemployment, of course. Results would be virtually identical absent the indexation of payments to average unemployment. The one difference is that absent indexation the relation in the left panel of Figure 1 would be an exact straight line by design.

[20]Table 5 further below provides the parameters associated with the optimal rule.

Figure 2: Impulse response to a recessionary productivity shock
—no federal RI and optimal federal RI—



Notes: Impulse responses to a one-standard-deviation negative productivity shock. Shown is the case of no federal RI (black solid lines) and the case with optimal federal RI (dashed blue lines). Impulse responses are derived under the assumption that the local policy instruments do not react at all to the introduction of a federal RI scheme. All variables are expressed in terms of percent deviation from the steady state (a "1" meaning the variable is 1% above the steady-state level). An exception are the response net transfers received and the production tax, which are both expressed in percent of output.

illustrates the fiscal mechanics. With labor-market instruments fixed, a rise in separation and a fall in hiring mean fiscal gains to the member state shortly after the shock materializes (owing to the layoff tax). Hence, to balance the budget, production taxes fall on impact. Eventually, however, as separation and hiring stabilize but unemployment remains high, the fiscal cost of unemployment benefits weighs on the budget, so that the production tax needs to rise at a time when the recession continues to persist.

The dashed blue lines, instead, show the case with the optimal federal RI scheme in place. With the optimal federal RI scheme, the labor-market response is sharply less pronounced. The separation rate no longer rises persistently after the shock. Unemployment rises be about an order of magnitude less than in the baseline. The key to this is the member state's inherent fiscal response as laid out in Section 3.1. Namely, the member state receives a sizable and persistent fiscal injection from the federal level in the midst of a recession. The injection induces cuts in taxes on production, $\tau_{J,t}$. The persistent cuts in taxes raise the surplus of firms. This stimulates hiring and reduces layoffs; all of this stabilizes employment and output, that is, it makes the recession in the member state less deep to start with.

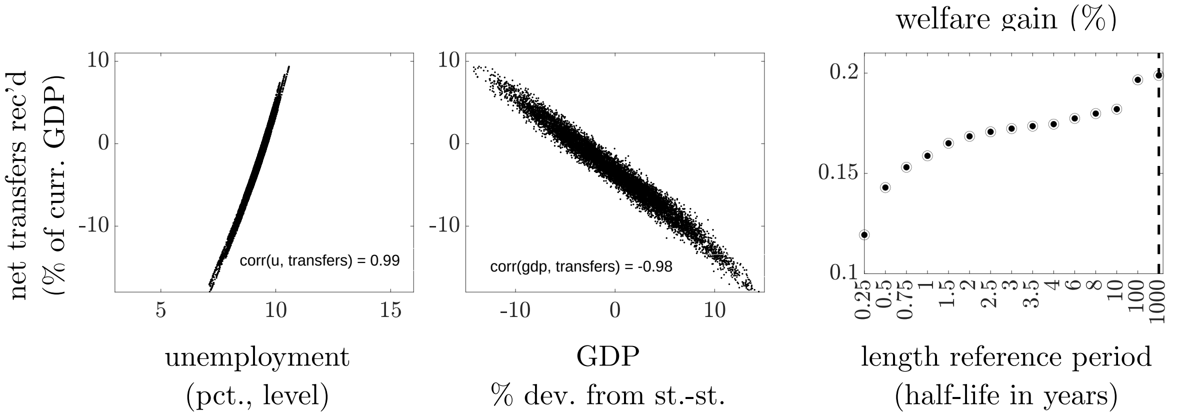
4.3.2 Optimal federal RI with optimizing member states–a long-run view

We will later argue that what limits the scope of federal RI is the transition period after federal RI is first introduced not the long-run incentives. To prepare the ground for this, therefore, this section abstracts from the transition period but it allows member states to adjust their policies.

Suppose that when designing the federal RI policy, the federal government looks at the long-run welfare gains only. Similarly, in choosing how to adjust its policy instruments in response to federal RI, the member state, too, only looks at the long run.[21] Figure 3 shows the implications for a federal RI that is optimized for this setting. The format is the same as in Figure 1. Indeed, the graphs in the two figures look virtually identical. Under the long-run view, the optimal federal RI scheme is about as generous as it would be absent any response by the member state. This is so even though the member states can adjust their labor-market policies. In the long run, indexation to past unemployment rates, thus, would serve its purpose: member states do not free-ride on the federal scheme.

[21] In that long run, in this setting, the federal scheme has to be self-financing period by period.

Figure 3: Optimal federal RI in the long run – without the transition phase



Notes: Same as Figure 1, with two differences. First, the member state optimizes all its labor-market policies after federal RI is introduced. Second, the member state and the federal level only look at welfare in the long run when making their policy choices. The welfare gains reported here also refer to those in the long run.

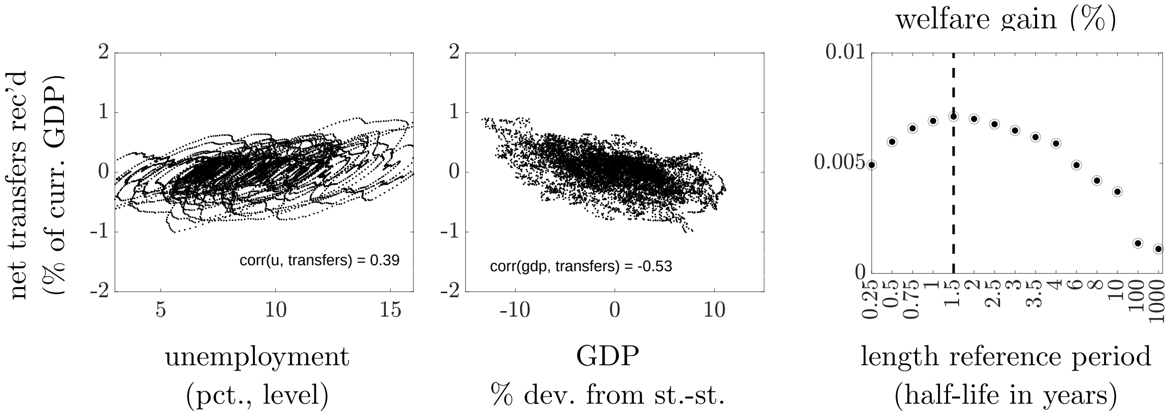
4.3.3 Optimal federal RI with optimizing states–accounting for the transition

This section shows that accounting for the incentives during the transition phase fundamentally changes the scope for and optimal form of federal RI. Optimal federal RI becomes less generous (a lower α) and less well-timed (a lower δ). With this, the welfare gains from federal RI are notably lower, too.

Figure 4 has the same format as the earlier Figures 1 and 3. What differs is the experiment. Now the member states can adjust their policies when the federal scheme is introduced *and* that they take into account not only the long run, but also what happens in the near term (the transition period).[22] The two left-most panels in Figure 4 show the transfers associated with the federal RI scheme that is optimal for this setting. Compared to the earlier scenarios, the maximal payouts are an order of magnitude smaller (reflecting a lower α). Still, federal RI does provide transfers, if at a lower level: on average, the optimal federal RI scheme here replaces somewhat more than three percent of the income lost to a recession. Note that transfers are lower while unemployment fluctuates more than in the previous two scenarios. The reason is simple: since federal RI transfers dampen unemployment fluctuations, lower transfers also mean less stabilization. The second important observation is that the payouts also are less well-timed. Rather than showing a nearly-

[22]The setting, thus, is exactly as described in Section 2.2 and the federal budget has to hold in present-value terms, see equation (23).

Figure 4: Optimal federal RI with the transition phase



Notes: Same as Figure 1, with the difference that the member state reoptimizes its labor-market policies after the federal RI scheme is introduced. The member state can adjust all local labor-market policies.

straight line, the two left-most panels show a cloud. The correlation between transfers and local unemployment in the optimal federal RI scheme now is 0.39. The correlation between GDP and transfers is -0.53. This weaker correlation of payouts and economic activity is easily explained. The optimal half-life of the reference period over which the usual unemployment rate is computed now falls to barely 1.5 years (a lower δ), see the vertical line in the right panel of Figure 4. This means that the reference level of unemployment catches up reasonably fast with actual unemployment. This deters the incentives to free-ride on the transition path. But it also means that unemployment can be lower than the reference level of unemployment at times when unemployment remains high (say, in the recovery phase of a recession), rendering federal transfers less well-timed. All this combined means that the welfare gains from optimal federal RI are an order of magnitude smaller than in the previous two scenarios. Taking into account the transition path, the welfare gains from optimal federal RI run to barely 0.0071 percent of life-time consumption.

4.3.4 Discussion

We have analyzed the scope for federal unemployment-based reinsurance when member states can adjust their labor-market policies after the federal transfer scheme is introduced. We found that member states' behavioral incentives during the transition period can greatly diminish the scope for federal transfers even though transfers are indexed to member states' own past unemployment level.

The mechanism behind these results is simple. Past average unemployment –by design– is backward-looking, not forward-looking. A member state that engineers higher-than-usual unemployment shortly after the federal RI scheme is introduced, therefore, all else equal acquires transfers from the federal RI scheme that it will never have to repay. This may be worthwhile for the member state even if though it means permanently-higher unemployment and even if, in the long run, the member state no longer receives transfers (since by then the reference level of unemployment will has caught up with the new reality). In equilibrium, a generous federal RI scheme would then mean higher unemployment in all the member states, and no fiscal gains (since federal RI taxes have to finance federal RI and rise accordingly). Anticipating this, the optimal federal RI scheme is less generous than absent the member states' behavioral response. In addition, the optimal federal RI scheme reduces the gains on the transition path by making the reference level of unemployment catch up faster with actual unemployment in the member state.
Table 5 summarizes the findings of Sections 4.3.1 through 4.3.3 in tabular form. The table reports on the federal and local policy choices in the respective scenario, on the implied change in average employment and on the standard deviation of employment. The table also reports the welfare gain associated with the respective scheme. The first three

Table 5: Summary results: optimal federal RI

	federal policy			Effect on local policy %			Effect on employm't (%)		welfare gain
Sect.	α	δ^*	$\boldsymbol{\tau}_F$	b	τ_ξ	τ_v	E	std	%
4.3.1	5.74	1000	-0.016	0	0	0	+2.0	- 82.8	.22
4.3.2	5.02	1000	0	3.55	1.35	-.23	+ .52	- 77.0	.20
4.3.3	0.16	1.5	-3e-5	0.62	-1.94	-.72	+ .04	- 2.0	.0071

Notes: For the scenarios discussed in Sections 4.3.1 through 4.3.3, the table reports the parameters of the optimal federal RI policy (first three columns) and the local governments' policy choices (next three columns). Shown are percent changes of the instrument, not percentage-point changes. Next, the table reports how the introduction of the federal scheme (and the member state's response) affects average employment (percent rise/fall) and the standard deviation of employment (percent rise/fall). The final column reports the welfare gain in consumption-equivalent percent. δ^* is the half-life implied by δ, in years.

columns report the parameters behind the optimal policy schemes. The differences in terms of generosity (α) and the half-life of the reference period (column titled δ^* in the table) have been discussed earlier. The tax $\boldsymbol{\tau}_F$ that finances the federal scheme need not be zero or positive since the scheme not only makes payouts but also asks for higher contributions from member states with low unemployment. What is more, federal RI stabilizes the business cycle. It therefore also induces higher average employment, owing to the well-known non-linearities in the search and matching model (Hairault et al. (2010), Jung and Kuester (2011), and Dupraz et al. (2019)). This reduces the financing needs for federal RI. In line with the previous discussion, the gains in stabilization and average employment that federal RI can achieve are much smaller when accounting for the transition period.

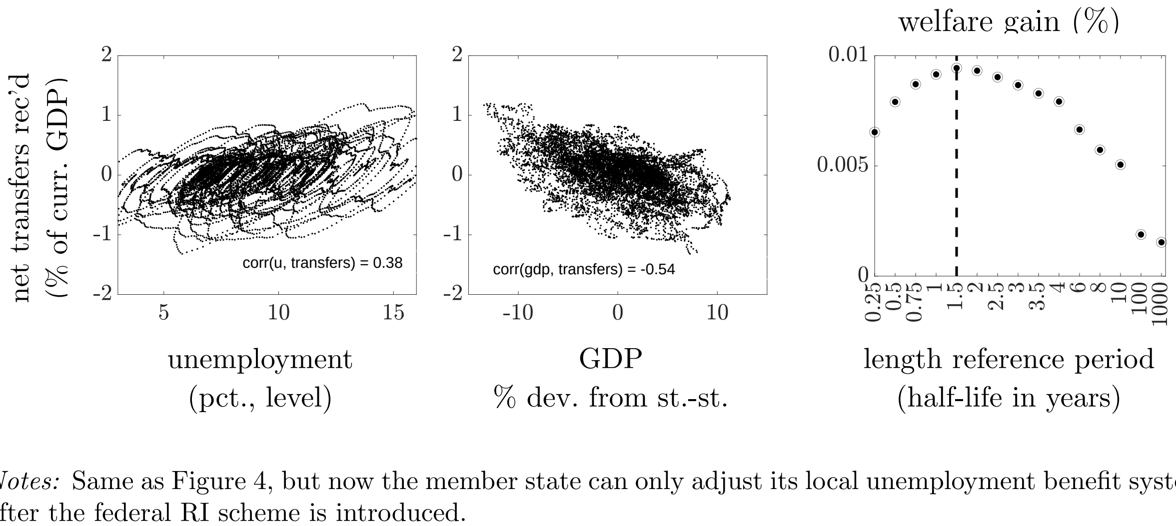
4.4 Sensitivity analysis

We have conducted a number of sensitivity checks, which we report here.

4.4.1 Limited number of instruments

We have run the same experiments as in Section 4.3 allowing the member state access to a limited number of instruments only. As an example, Figure 5 shows the case when the member state can adjust unemployment benefits only. The welfare gains are slightly larger and federal RI slightly more generous, replacing on average somewhat more than four percent of the income lost to the recession. The reference period for unemployment remains short. The welfare gains remained comparably small when we allowed the member

Figure 5: Optimal federal RI with transition–only unemployment benefits adjust



Notes: Same as Figure 4, but now the member state can only adjust its local unemployment benefit system after the federal RI scheme is introduced.

state to only adjust layoff taxes and hiring subsidies, keeping unemployment benefits at their pre-federal-RI level. To us, this case seems important, for it shows that harmonizing some labor-market policies (such as the unemployment benefit system) but not others does not by itself render a more generous federal transfer scheme viable.

4.4.2 Changing the numerical implementation

The baseline results above were produced using a fourth-order perturbation. To see if this matters, we ran a subset of scenarios using a third-order approximation only. The computed welfare gains of optimal federal RI were somewhat lower still in this sceneario than using the fourth-order approximation. The pattern in terms of the economics were identical, however (optimal federal RI continued to have short reference periods, for example).
Next, one may be concerned that the optimization routines that are part of the algorithm detect local maxima only. To check the sensitivity of the results with respect to this, we also ran a subset of the scenarios on a fixed grid for α. The results were not affected.

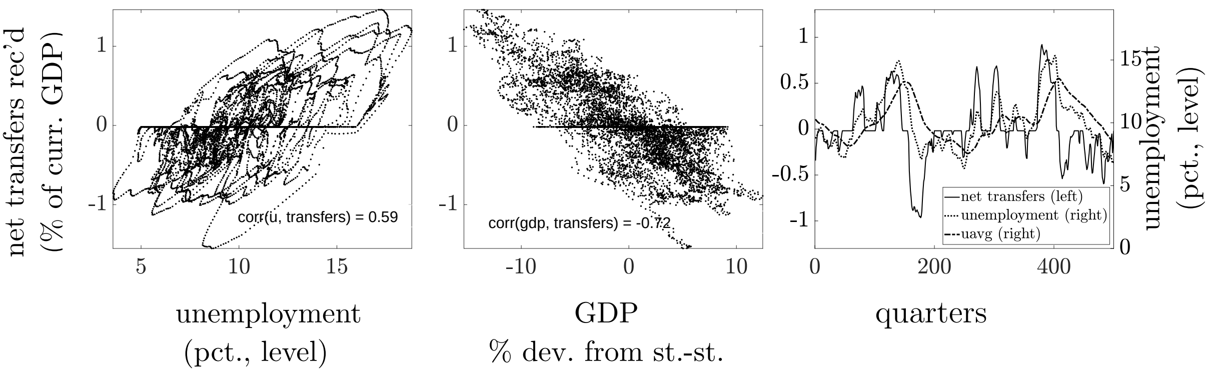
4.5 Optimal federal RI with an unemployment threshold

The current section looks at the optimal generosity of federal RI when the scheme features a threshold, that is, when the federal level only pays if unemployment exceeds its reference level by a certain margin. We do so to approximate the federal/state funding structure of the US unemployment insurance system where the federal level contributes to the costs of unemployment insurance only in deep-enough recessions. A threshold means that federal RI no longer provides transfers that help stabilize the "normal" business cycle. At the same time, the threshold serves to somewhat mitigate member states' moral hazard during the transition phase.
As discussed in Section 4.2, such a setup renders the policy problem notably more computationally-demanding. Here, therefore, we look at just one policy exercise that can serve as indicative. Namely, we fix the threshold in equation (22) to $\Phi = 0.015$. This means that the federal RI scheme will make transfers whenever unemployment is 1.5 percentage points (or, half a standard deviation or, roughly, 15 percent) above its historical average. Otherwise, we set up the policy experiment in a way that is comparable to that shown in Figure 5. Namely, we allow the member state to optimize unemployment benefits only. Absent a threshold, very low half-lives of the reference period serve the purpose of preventing member states' moral hazard during the transition phase. This purpose now can be served partly by the threshold. We fix parameter $\delta = 0.9857$, so that the reference

level of unemployment has a reasonably long half-life of four years.[23]

Figure 6 provides a graphical summary of what the optimal federal RI scheme in this class implies. The panel on the right shows simulated time series of the evolution of

Figure 6: Optimal federal RI–a threshold scheme



Notes: The policy experiment is the same as in Figure 5, with the one difference being that the federal RI scheme here features a threshold at $\Phi = 0.015$. The member state can only adjust unemployment benefits. The two left-most panels have a design that is identical to the setup in the earlier figures. The panel on the right plots net transfers (left axis, solid line, percent of GDP), against a time series of unemployment and the reference level of unemployment (right axis, dotted and dashed-dotted, respectively).

transfers against unemployment. When unemployment rises steeply (dotted line), transfers rise (solid line). At the same time, persistent recessions mean that the reference level of unemployment (dashed-dotted line) also rises. This means that in the recovery phase the withdrawal rate of federal transfers can be steep.

The two left-most panels show the generosity of federal RI and the stabilization benefits. These panels are identical in design to the two left-most panels of the earlier figures. Whenever it makes payouts, the federal RI scheme can be more generous than absent the threshold, replacing here about 10% of the income lost to a severe recession. The very nature of having a threshold, however, means that there is no fiscal risk sharing for smaller shocks. This explains the horizontal lines of zero transfers in the two panels. On average, across all possible states of nature, the federal scheme replaces about 6.5 percent of the income lost to a recession.[24] Overall, the stabilization gains are not notably larger than in Figure 5. At a consumption-equivalent welfare gain of 0.016 percent of life-time

[23]The optimal α the federal planner uses is 0.275 and the member state sets benefits at $b = .3717$, roughly 2.7% larger than in autarky. The details of the algorithm is described in Appendix C.

[24]While these numbers appear small, note that this a notable part of the extent of fiscal risk sharing that the literature finds for the US, for example. Feyrer and Sacerdote (2013) and Asdrubali et al. (1996),

consumption, the federal RI scheme with a threshold can realize about 7 percent of the welfare gains that could be possible if member state's incentives were under control.

5 Conclusions

What is the scope of a federal unemployment reinsurance scheme (RI) in a union of member states that retain authority over local labor-market policies? Such a scheme provides transfers to member states in recessions, that is, at a time when unemployment is high and the fiscal budget burdened. In the paper's setup, these transfers alleviate the fall in consumption and shorten the length of the recession itself. At the same time, the fiscal transfers raise concerns of moral hazard by the member state. The paper has provided theory and a quantitative exploration for a stylized European Monetary Union.

We looked at transfer mechanisms that index payouts to the member states' unemployment experience and, thus, prevent permanent free riding. The main finding was that, nevertheless, the member states' moral hazard constrains the scope for federal RI. This has to do with the transition phase after the federal fiscal capacity is introduced (the "near term") in which federal transfers induce the member state to introduce a less employment-friendly fiscal mix. Depending on the specific scenario that we looked at and the policy choices that we gave to member states, the optimized federal RI schemes replaces between 3 and 7 percent of the income that a member state loses to the recession. That is, even accounting for member states' moral hazard there remains scope for transfers, but the transfers optimally were short-lived.

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for example, estimate that in the short term between 13 and 25 cents of every dollar of a state-level income shock is offset by federal fiscal policy.

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— Online Appendix —

A Proofs

This appendix provides the proofs for the proposition and the corollary in the main text. The outline of the proofs builds heavily on Jung and Kuester (2015), the results of which we extend to the case of federal unemployment reinsurance. The proofs proceeds in steps. Appendix A.1 provides the planner's problem for the member state. Appendix A.2 provides a number of useful transformations that are handy later on. Appendix A.3 describes the decentralized equilibrium. Appendix A.4 decentralizes the planner's allocation in steady state and (if cyclical instruments are allowed) also over the business cycle. Building on this, Section A.5 proves the main text's proposition. Section A.6 defines the microelasticity used in the main text's corollary. The proof of the corollary is in Section A.7.

A.1 Planner's problem in the member state

We state the social planner's problem in recursive form. The planner enters the period facing the following state variables: aggregate productivity state a, e^p employed workers and a promised utility difference Δ^p. Here as in the following, a superscript ^{p} marks the allocation in the planner problem. The planner maximizes a utilitarian welfare function, choosing state-contingent promised utility $\Delta^{p\prime}_{a'}$,[25] making consumption choices c^p_e and c^p_u, separation decisions ξ^p and choosing market tightness θ^p. Letting $\Psi_x(x)$ denote the option value of having a choice with $\Psi_x(x) := -\psi_x[(1-x)\log(1-x) + x\log(x)]$, and with μ_ϵ the average cost.

The member-state planner faces a federal unemployment-based reinsurance system $B_F(u^p) - \tau_F$ where $B_F(1-e^p)$ is a general function in unemployment and τ_F a constant payment towards Europe.

The planner's problem can be written as

$$\mathtt{W}(a, e^p, \Delta^p) = \max_{\xi^p,\theta^p,c^p_e,c^p_u,\{\Delta^{p\prime}_{a'}\}} e^p\mathtt{u}(c^p_e)+(1-e^p)\mathtt{u}(c^p_u)+[\xi^p e^p+(1-e^p)](\Psi_s(s^p)+\overline{h})+\beta\mathbb{E}_a\mathtt{W}(a', e^{p\prime}, \Delta^{p\prime})$$

subject to the *budget constraint*

$$\begin{aligned} & e^p(1-\xi^p)\exp\{a\} + B_F(1-e^p) - \tau_F \\ = \; & c^p_e e^p + (1-e^p)c^p_u + \mu_\epsilon(1-\xi^p)e^p - e^p\Psi_\xi(\xi^p) + \kappa_v[\xi^p e^p + (1-e^p)]s^p\theta^p, \end{aligned} \tag{36}$$

the *participation constraint*

$$s^p = \frac{1}{1 + e^{\frac{-f^p\beta\mathbb{E}_a\Delta^{p\prime}}{\psi_s}}} \tag{37}$$

the *promise-keeping constraint*

$$\Delta^p = \mathtt{u}(c^p_e) - \overline{h}(1-\xi^p) - \mathtt{u}(c^p_u) + \beta E_a\Delta^{p\prime}(1-\xi^p) + (1-\xi^p)\psi_s\log(1-s^p), \tag{38}$$

[25] In terms of notation, subindex $_{a'}$ here indicates that the level of future promised utility is chosen in a state-contingent way.

and the *constraint on the law of motion for employment*

$$e^{p\prime} = e^p(1-\xi^p) + [\xi^p e^p + (1-e^p)] s^p \chi\left(\theta^p\right)^\gamma . \tag{39}$$

The last expression uses that the job-finding rate is defined as $f \equiv \chi\left(\theta^p\right)^\gamma$. Denoting by λ^p_Δ the Lagrange multiplier on the promise keeping constraint and by λ^p_c the Pagrange multiplier on the budget constraint, the first-order conditions with respect to the two consumption levels c_e and c_u deliver:

$$\lambda^p_\Delta = \frac{[\mathtt{u}'(c^p_e) - \mathtt{u}'(c^p_u)] e^p (1-e^p)}{\mathtt{u}'(c^p_e)(1-e^p) + e^p \mathtt{u}'(c^p_u)}, \tag{40}$$

$$\lambda^p_c = \frac{\mathtt{u}'(c^p_e)\mathtt{u}'(c^p_u)}{\mathtt{u}'(c^p_e)(1-e^p) + e^p \mathtt{u}'(c^p_u)} = \left[\frac{e^p}{\mathtt{u}'(c_e)} + \frac{1-e^p}{\mathtt{u}'(c_u)}\right]^{-1}. \tag{41}$$

The first-order condition regarding separations, ξ^p, is given by

$$\begin{aligned} 0 = \; & [\Psi_s(s) + \overline{h}] + \lambda^p_c[-\exp\{a\} + \mu_\epsilon + \Psi'_\xi(\xi^p) - \kappa_v s^p \theta^p] \\ & + \tfrac{\lambda^p_\Delta}{e^p}[\mathbb{E}_a \beta \Delta^{p\prime} - \overline{h}] + \tfrac{\lambda^p_\Delta}{e^p} \psi_s \log(1-s) + \beta \mathbb{E}_a \tfrac{\partial \mathtt{W}}{\partial e'}[-1 + s f^p]. \end{aligned} \tag{42}$$

Dividing through by λ^p_c we obtain

$$\begin{aligned} 0 = \; & \tfrac{\Psi_s(s)+\overline{h}}{\lambda^p_c} + [-\exp\{a\} + \mu_\epsilon + \Psi'_\xi(\xi^p) - \kappa_v s^p \theta^p] + \tfrac{\lambda^p_\Delta}{\lambda^p_c e}[\mathbb{E}_a \beta \Delta' - \overline{h}] \\ & + \tfrac{\lambda^p_\Delta}{\lambda_c e} \psi_s \log(1-s) + \beta \mathbb{E}_a \tfrac{\lambda^{p\prime}_c}{\lambda^p_c} \tfrac{\frac{\partial \mathtt{W}}{\partial e'}}{\lambda'_c}[-1 + s f^p]. \end{aligned}$$

This can be simplified further. First, observe that

$$s = \frac{1}{1 + \exp\{-f^p \beta \mathbb{E}_a \Delta^{p\prime}/\psi_s\}}.$$

So $\psi_s \log((1-s)/s) = -f^p \beta \mathbb{E}_a \Delta^{p\prime}$. Also observe that

$$-\psi_s[(1-s)]\log(1-s) + s\log(s)] = -\psi_s \log(1-s) + \psi_s \log\frac{1-s}{s} = -\psi_s \log(1-s) - s f^p \beta \mathbb{E}_a \Delta^{p\prime}.$$

Second, observe that

$$\frac{\lambda_\Delta}{\lambda^p_c e} = \left[\frac{1}{\mathtt{u}'(c^p_u)} - \frac{1}{\mathtt{u}'(c^p_e)}\right](1-e^p), \text{ and}$$

$$\frac{1}{\lambda^p_c} - \frac{1}{\mathtt{u}'(c^p_e)} = \frac{\mathtt{u}'(c^p_e)(1-e^p) + e^p \mathtt{u}'(c^p_u)}{\mathtt{u}'(c^p_e)\mathtt{u}'(c^p_u)} - \frac{1}{\mathtt{u}'(c^p_e)} = \frac{1-e^p}{\mathtt{u}'(c^p_u)} - \frac{1-e^p}{\mathtt{u}'(c^p_e)} = \frac{\lambda^p_\Delta}{\lambda^p_c e}.$$

Using these steps, the first-order condition for separations can be written as

$$
\begin{aligned}
0 \quad = & \quad \frac{-\psi_s \log(1-s) - s f^p \beta \Delta' + \overline{h}}{\lambda_c^p} + [-\exp\{a\} + \mu_\epsilon + \Psi'_\xi(\xi^p) - \kappa_v s^p \theta^p] \\
& + \left[\frac{1}{\lambda_c^p} - \frac{1}{\mathtt{u}'(c_e)}\right][\mathbb{E}_a \beta \Delta^{p\prime} - \overline{h}] + \left[\frac{1}{\lambda_c^p} - \frac{1}{\mathtt{u}'(c_e)}\right]\psi_s \log(1-s) + \beta \mathbb{E}_a \frac{\lambda_c^{p\prime}}{\lambda_c^p}\frac{\frac{\partial \mathtt{W}}{\partial e'}}{\lambda_c^{p\prime}}[-1 + s f^p].
\end{aligned}
$$

Rearranging delivers:

$$
\begin{aligned}
0 \quad = & \quad [-\exp\{a\} + \mu_\epsilon + \Psi'_\xi(\xi^p) - \kappa_v s^p \theta^p] - \frac{1}{\mathtt{u}'(c_e^p)}[\mathbb{E}_a \beta \Delta' - \overline{h}] - \frac{1}{\mathtt{u}'(c_e^p)}\psi_s \log(1-s) \\
& - \beta \mathbb{E}_a \frac{\lambda_c^{p\prime}}{\lambda_c^p}\left[\frac{\frac{\partial \mathtt{W}}{\partial e'}}{\lambda_c^{p\prime}} - \frac{\Delta'}{\lambda_c^{p\prime}}\right][1 - s f^p]. \qquad (43)
\end{aligned}
$$

Using $\Psi'_\xi(\xi^p) = \psi_\epsilon \log((1-\xi^p)/\xi^p)$ we can rearrange further to

$$
\xi^p = \frac{1}{1 + \exp\left\{\frac{\exp\{a\} - \mu_\epsilon + \kappa_v s^p \theta^p + \mathbb{E}_a \beta \frac{\lambda_c^{p\prime}}{\lambda_c^p}\left[\frac{\frac{\partial \mathtt{W}}{\partial e^{p\prime}}}{\lambda_c^{p\prime}} - \frac{\Delta^{p\prime}}{\lambda_c^{p\prime}}\right](1 - s^p f^p) + \frac{1}{\mathtt{u}'(c_e^p)}\left[\mathbb{E}_a \beta \Delta^{p\prime} - \overline{h}\right] + \frac{1}{\mathtt{u}'(c_e^p)}\psi_s \log(1 - s^p)}{\psi_\epsilon}\right\}}. \qquad (44)
$$

The first-order condition for market-tightness θ delivers:

$$
\begin{aligned}
0 \quad = & \quad -\left[\kappa_v + \kappa_v \frac{\partial s^p}{\partial \theta^p}\frac{\theta^p}{s^p}\right] - \mathbb{E}_a \beta \frac{\Delta^{p\prime}}{\lambda_c^p}\frac{\partial s^p}{\partial \theta^p}\frac{f^p}{s^p} + \mathbb{E}_a \beta \frac{\lambda_c^{p\prime}}{\lambda_c^p}\frac{\frac{\partial \mathtt{W}}{\partial e^{p\prime}}}{\lambda_c^{p\prime}}\left[\gamma \frac{f^p}{\theta^p} + \frac{\partial s^p}{\partial \theta^p}\frac{f^p}{s^p}\right] \\
& + \frac{\lambda_\Delta^p}{\lambda_c^p}\psi_s \frac{1}{1 - s^p}\frac{\partial s^p}{\partial \theta^p}\frac{1}{s^p}\frac{(1-\xi^p)}{[\xi^p e^p + (1 - e^p)]}. \qquad (45)
\end{aligned}
$$

The first-order conditions for state-contingent promised utility $\Delta^{p\prime}$ are:

$$
\begin{aligned}
\beta \frac{\lambda_\Delta^p}{\lambda_c^p}(1-\xi^p) \quad = & \quad \beta \frac{\lambda_c^{p\prime}}{\lambda_c^p}\frac{\frac{\partial \mathtt{W}}{\partial \Delta^{p\prime}}}{\lambda_c^{p\prime}} + \frac{\lambda_\Delta^p}{\lambda_c^p}\frac{\psi_s}{1 - s^p}(1-\xi^p)\frac{\partial s^p}{\partial \Delta^{p\prime}} \\
& + [1 - (1-\xi^p)e^p]\psi_s \log\left(\frac{1 - s^p}{s^p}\right)\frac{\partial s^p}{\partial \Delta^{p\prime}} \\
& + \mathbb{E}_a \beta \frac{\lambda_c^{p\prime}}{\lambda_c^p}\frac{\frac{\partial \mathtt{W}}{\partial e^{p\prime}}}{\lambda_c^{p\prime}} f^p \frac{\partial s^p}{\partial \Delta^{p\prime}}[1 - (1-\xi^p)e^p] - \kappa_v \frac{\partial s^p}{\partial \Delta^{p\prime}}\theta^p[1 - (1-\xi^p)e^p].
\end{aligned}
$$

Using the participation constraint (37) to substitute out for $\psi_s \log\left(\frac{1-s^p}{s^p}\right)$, this can be further simplified to yield

$$
\begin{aligned}
\beta \frac{\lambda_\Delta^p}{\lambda_c^p}(1-\xi^p) \quad = \quad & \beta \frac{\lambda_c^{p\prime}}{\lambda_c^p} \frac{\frac{\partial \mathtt{W}}{\partial \Delta^{p\prime}}}{\lambda_c^{p\prime}} + \frac{\lambda_\Delta^p}{\lambda_c^p} \frac{\psi_s}{1-s^p}(1-\xi^p) \frac{\partial s^p}{\partial \Delta^{p\prime}} \\
& -[1-(1-\xi^p)e^p] f^p \beta \mathbb{E}_a \left\{ \frac{\Delta^{p\prime}}{\lambda_c^p} \right\} \frac{\partial s^p}{\partial \Delta^{p\prime}} \\
& +\mathbb{E}_a \beta \frac{\lambda_c^{p\prime}}{\lambda_c^p} \frac{\frac{\partial \mathtt{W}}{\partial e^{p\prime}}}{\lambda_c^{p\prime}} f^p \frac{\partial s^p}{\partial \Delta^{p\prime}} [1-(1-\xi^p)e^p] \\
& -\kappa_v \frac{\partial s^p}{\partial \Delta^{p\prime}} \theta^p [1-(1-\xi^p)e^p]. \qquad (46)
\end{aligned}
$$

The *envelope conditions* are:

$$
\frac{\frac{\partial \mathtt{W}}{\partial \Delta^p}}{\lambda_c^p} = \frac{\lambda_\Delta^p}{\lambda_c^p}. \qquad (47)
$$

$$
\begin{aligned}
\frac{\frac{\partial \mathtt{W}}{\partial e^p}}{\lambda_c^p} = \; & \frac{[\mathtt{u}(c_e^p) - \mathtt{u}(c_u^p)]}{\lambda_c^p} - \frac{(1-\xi^p)\left[\Psi(s^p) + \overline{h}\right]}{\lambda_c^p} + \mathbb{E}_a \beta \frac{\lambda_c^{p\prime}}{\lambda_c^p} \frac{\frac{\partial \mathtt{W}}{\partial e^{p\prime}}}{\lambda_c^{p\prime}} (1-s^p f^p)(1-\xi^p) \\
& +(1-\xi^p)\left[\exp\{a\} - \mathbf{B}_F^\prime(u_t^p)/(1-\xi) - \mu_\epsilon\right] - c_e^p + c_u^p + \Psi_\xi(\xi^p) \\
& +\kappa_v s^p \theta^p (1-\xi^p). \qquad (48)
\end{aligned}
$$

A.2 A number of useful transformations

Here we present a number of algebraic transformations that will be useful for proving how to decentralize the allocation in the planner's problem.

A.2.1 Rewriting envelope condition (48)

We start by rewriting envelope condition (48) as (49): To ease the burden on notation later on, define $J^p := \frac{\frac{\partial \mathtt{W}}{\partial e^p}}{\lambda_c^p} - \frac{\Delta^p}{\lambda_c^p}$. We show that

$$
\begin{aligned}
J^p \quad = \quad & \exp\{a\} - \mathbf{B}_F^\prime(u_t^p) - \mu_\epsilon - c_e^p + c_u^p - \psi_\epsilon \log(1-\xi^p) \\
& +\kappa_v s^p \theta^p + \frac{\xi^p}{\mathtt{u}^\prime(c_e^p)}\left[\mathbb{E}_a \beta \Delta^{p\prime} - \overline{h}\right] + \frac{\xi^p}{\mathtt{u}^\prime(c_e^p)} \psi_s \log(1-s^p) \\
& +\mathbb{E}_a \beta \frac{\lambda_c^{p\prime}}{\lambda_c^p} J^{p\prime}(1-s^p f^p). \qquad (49)
\end{aligned}
$$

To see that this is true, first observe that we can rewrite

$$
\Psi_\xi(\xi^p) := -\psi_\epsilon \left[(1-\xi^p)\log(1-\xi^p) + \xi^p \log(\xi^p)\right]
$$

since $\Psi_\xi(\xi^p) = -\psi_\epsilon \log(1-\xi^p) + \psi_\epsilon \xi^p \log(\frac{1-\xi^p}{\xi^p})$. Taking the derivative with respect to ξ^p, we have

$$\Psi'_\xi(\xi^p) = -\psi_\epsilon \log(\xi^p/(1-\xi^p)).$$

Define ϵ^{ξ^p} via

$$\xi^p = Prob\left(\epsilon_j \geq \epsilon^{\xi^p}\right) = 1 - 1/\left(1 + \exp\left\{-\left(\epsilon^{\xi^p} - \mu_\epsilon\right)/\psi_\epsilon\right\}\right), \tag{50}$$

so that

$$\epsilon^{\xi^p} - \mu_\epsilon = -\psi_\epsilon \log(\xi^p/(1-\xi^p))$$

With this, we have that

$$\Psi'_\xi(\xi^p) = \epsilon^{\xi^p} - \mu_\epsilon.$$

Also, from above, observe that

$$\Psi_\xi(\xi^p) = -\psi \log(1-\xi^p) + \xi^p(\epsilon^{\xi^p} - \mu_\epsilon). \tag{51}$$

Use the formula for Ψ'_ξ in the planner's first-order condition for separations (43), to get

$$\begin{aligned} \epsilon^{\xi^p} \quad = \quad & \exp\{a\} + \kappa_v s^p \theta^p + \mathbb{E}_a \beta \frac{\lambda_c^{p\prime}}{\lambda_c^p}\left[\frac{\frac{\partial \mathtt{W}}{\partial e^{p\prime}}}{\lambda_c^{p\prime}} - \frac{\Delta^{p\prime}}{\lambda_c^{p\prime}}\right](1 - s^p f^p) + \frac{1}{\mathtt{u}'(c_e^p)}\left[\mathbb{E}_a \beta \Delta^{p\prime} - \overline{h}\right] \\ & + \frac{1}{\mathtt{u}'(c_e^p)}\psi_s \log(1-s^p). \end{aligned} \tag{52}$$

Substituting for Ψ_ξ from (51) in the planner problem's envelope condition, (48), gives

$$\begin{aligned} \frac{\frac{\partial \mathtt{W}}{\partial e^p}}{\lambda_c^p} \quad = \quad & \frac{[\mathtt{u}(c_e^p) - \mathtt{u}(c_u^p)]}{\lambda_c^p} - \frac{(1-\xi^p)\left[\Psi(s^p) + \bar{h}\right]}{\lambda_c^p} + \mathbb{E}_a \beta \frac{\lambda_c^{p\prime}}{\lambda_c^p} \frac{\frac{\partial \mathtt{W}}{\partial e^{p\prime}}}{\lambda_c^{p\prime}}(1 - s^p f^p)(1-\xi^p) \\ & + (1-\xi^p)\left[\exp\{a\} - \mathbf{B}'_F(u_t^p)/(1-\xi) - \mu_\epsilon\right] - c_e^p + c_u^p - \psi \log(1-\xi^p) + \kappa_v s^p \theta^p (1-\xi^p) \\ & + \xi^p(\epsilon^{\xi^p} - \mu_\epsilon). \end{aligned}$$

Use (52) to get

$$\begin{aligned} \frac{\frac{\partial \mathtt{W}}{\partial e^p}}{\lambda_c^p} \quad = \quad & \frac{[\mathtt{u}(c_e^p) - \mathtt{u}(c_u^p)]}{\lambda_c^p} - \frac{(1-\xi^p)\left[\Psi(s^p) + \bar{h}\right]}{\lambda_c^p} + \mathbb{E}_a \beta \frac{\lambda_c^{p\prime}}{\lambda_c^p} \frac{\frac{\partial \mathtt{W}}{\partial e^{p\prime}}}{\lambda_c^{p\prime}}(1 - s^p f^p)(1-\xi^p) \\ & + \exp\{a\} - \mu_\epsilon - \mathbf{B}'_F(u_t^p) - c_e^p + c_u^p - \psi_\epsilon \log(1-\xi^p) + \kappa_v s^p \theta^p \\ & + \xi_p\left[\mathbb{E}_a \beta \frac{\lambda_c^{p\prime}}{\lambda_c^p}\left[\frac{\frac{\partial \mathtt{W}}{\partial e^{p\prime}}}{\lambda_c^{p\prime}} - \frac{\Delta^{p\prime}}{\lambda_c^{p\prime}}\right](1 - s^p f^p) + \frac{1}{\mathtt{u}'(c_e^p)}\left[\mathbb{E}_a \beta \Delta^{p\prime} - \overline{h}\right] + \frac{1}{\mathtt{u}'(c_e^p)}\psi_s \log(1-s^p)\right]. \end{aligned}$$

Next, substitute for $\Psi(s^p) = -\psi_s \log(1-s^p) + \psi_s s^p \log \frac{1-s^p}{s^p} = -\psi_s \log(1-s^p) - s^p f^p \beta \mathbb{E}_a \Delta^{p\prime}$, where the last step, again, uses the participation constraint, equation (37). This gives

$$\begin{aligned}\frac{\frac{\partial \mathtt{W}}{\partial e^p}}{\lambda_c^p} &= \frac{\mathtt{u}(c_e^p)-\mathtt{u}(c_u^p)-\overline{h}(1-\xi^p)+(1-\xi^p)\psi_s\log(1-s^p)}{\lambda_c^p}+\frac{(1-\xi^p)s^pf^p\beta\mathbb{E}_a\Delta^{p\prime}}{\lambda_c^p}\\ &+\mathbb{E}_a\beta\frac{\lambda_c^{p\prime}}{\lambda_c^p}\frac{\frac{\partial \mathtt{W}}{\partial e^{p\prime}}}{\lambda_c^{p\prime}}(1-s^pf^p)(1-\xi^p)\\ &+\exp\{a\}-\mathbf{B}_F'(u_t^p)-\mu_\epsilon-c_e^p+c_u^p-\psi_\epsilon\log(1-\xi^p)\\ &+\kappa_v s^p\theta^p+\frac{\xi^p}{\mathtt{u}'(c_e^p)}\left[\mathbb{E}_a\beta\Delta^{p\prime}-\overline{h}\right]+\frac{\xi^p}{\mathtt{u}'(c_e^p)}\psi_s\log(1-s^p)\\ &+\xi^p\,\mathbb{E}_a\beta\frac{\lambda_c^{p\prime}}{\lambda_c^p}\left[\frac{\frac{\partial \mathtt{W}}{\partial e^{p\prime}}}{\lambda_c^{p\prime}}-\frac{\Delta^{p\prime}}{\lambda_c^{p\prime}}\right](1-s^pf^p).\end{aligned}$$

Finally, from the promise-keeping constraint, equation (38), observe that

$$\frac{\mathtt{u}(c_e^p)-\overline{h}(1-\xi^p)-\mathtt{u}(c_u^p)+(1-\xi^p)\psi_s\log(1-s^p)}{\lambda_c^p}=\frac{\Delta^p}{\lambda_c^p}-\frac{\beta E_a\Delta^{p\prime}(1-\xi^p)}{\lambda_c^p}.$$

Substituting this into the envelope condition, we have:

$$\begin{aligned}\frac{\frac{\partial \mathtt{W}}{\partial e^p}}{\lambda_c^p} &= \frac{\Delta^p}{\lambda_c^p}-\frac{\beta E_a\Delta^{p\prime}(1-\xi^p)}{\lambda_c^p}+\frac{(1-\xi^p)s^pf^p\beta\mathbb{E}_a\Delta^{p\prime}}{\lambda_c^p}\\ &+\mathbb{E}_a\beta\frac{\lambda_c^{p\prime}}{\lambda_c^p}\frac{\frac{\partial \mathtt{W}}{\partial e^{p\prime}}}{\lambda_c^{p\prime}}(1-s^pf^p)(1-\xi^p)\\ &+\exp\{a\}-\mathbf{B}_F'(u_t^p)-\mu_\epsilon-c_e^p+c_u^p-\psi_\epsilon\log(1-\xi^p)\\ &+\kappa_v s^p\theta^p+\frac{\xi^p}{\mathtt{u}'(c_e^p)}\left[\mathbb{E}_a\beta\Delta^{p\prime}-\overline{h}\right]+\frac{\xi^p}{\mathtt{u}'(c_e^p)}\psi_s\log(1-s^p)\\ &+\xi^p\,\mathbb{E}_a\beta\frac{\lambda_c^{p\prime}}{\lambda_c^p}\left[\frac{\frac{\partial \mathtt{W}}{\partial e^{p\prime}}}{\lambda_c^{p\prime}}-\frac{\Delta^{p\prime}}{\lambda_c^{p\prime}}\right](1-s^pf^p).\end{aligned}$$

Bring $\frac{\Delta^p}{\lambda_c^p}$ to the left-hand side and rearrange. This gives

$$\begin{aligned}\frac{\frac{\partial \mathtt{W}}{\partial e^p}}{\lambda_c^p}-\frac{\Delta^p}{\lambda_c^p} &= \exp\{a\}-\mathbf{B}_F'(u_t^p)-\mu_\epsilon-c_e^p+c_u^p-\psi_\epsilon\log(1-\xi^p)\\ &+\kappa_v s^p\theta^p+\frac{\xi^p}{\mathtt{u}'(c_e^p)}\left[\mathbb{E}_a\beta\Delta^{p\prime}-\overline{h}\right]+\frac{\xi^p}{\mathtt{u}'(c_e^p)}\psi_s\log(1-s^p)\\ &+\mathbb{E}_a\beta\frac{\lambda_c^{p\prime}}{\lambda_c^p}\left[\frac{\frac{\partial \mathtt{W}}{\partial e^{p\prime}}}{\lambda_c^{p\prime}}-\frac{\Delta^{p\prime}}{\lambda_c^{p\prime}}\right](1-s^pf^p).\end{aligned}$$

To ease the burden on notation later on, define $J^p := \frac{\frac{\partial \mathtt{W}}{\partial e^p}}{\lambda_c^p} - \frac{\Delta^p}{\lambda_c^p}$, so

$$\begin{aligned} J^p \quad = & \quad \exp\{a\} - \mathbf{B}_F'(u_t^p) - \mu_\epsilon - c_e^p + c_u^p - \psi_\epsilon \log(1-\xi^p) \\ & +\kappa_v s^p \theta^p + \frac{\xi^p}{\mathtt{u}'(c_e^p)} \left[\mathbb{E}_a \beta \Delta^{p\prime} - \overline{h}\right] + \frac{\xi^p}{\mathtt{u}'(c_e^p)} \psi_s \log(1-s^p) \\ & +\mathbb{E}_a \beta \frac{\lambda_c^{p\prime}}{\lambda_c^p} J^{p\prime}(1 - s^p f^p). \end{aligned} \tag{53}$$

Here J^p can be interpreted as the profit equation of the planner.

A.2.2 Simplify J^p further

Also for later use in the proofs, we next substitute further. Use (51) and (52) in the budget constraint (36). to get

$$\begin{aligned} & e^p(1-\xi^p)\exp\{a\} + B_F(1-e^p) - \tau_F \\ = \; & c_e^p e^p + (1-e^p) c_u^p + \mu_\epsilon (1-\xi^p) e^p \\ & +\kappa_v [\xi^p e^p + (1-e^p)] s^p \theta^p, \\ & +e^p \psi \log(1-\xi^p) \\ & -e^p \xi^p (\exp\{a\} + \kappa_v s^p \theta^p + \mathbb{E}_a \beta \frac{\lambda_c^{p\prime}}{\lambda_c^p} \left[\frac{\frac{\partial \mathtt{W}}{\partial e^{p\prime}}}{\lambda_c^{p\prime}} - \frac{\Delta^{p\prime}}{\lambda_c^{p\prime}} \right] (1 - s^p f^p) \\ & + \frac{1}{\mathtt{u}'(c_e^p)} \left[\mathbb{E}_a \beta \Delta^{p\prime} - \overline{h}\right] + \frac{1}{\mathtt{u}'(c_e^p)} \psi_s \log(1-s^p). - \mu_\epsilon). \end{aligned} \tag{54}$$

$$\begin{aligned} & e^p \exp\{a\} + B_F(1-e^p) - \tau_F \\ = \; & c_e^p e^p + (1-e^p) c_u^p + \mu_\epsilon e^p \\ & +\kappa_v [(1-e^p)] s^p \theta^p, \\ & +e^p \psi_\epsilon \log(1-\xi^p) \\ & -e^p \xi^p (\mathbb{E}_a \beta \frac{\lambda_c^{p\prime}}{\lambda_c^p} \left[\frac{\frac{\partial \mathtt{W}}{\partial e^{p\prime}}}{\lambda_c^{p\prime}} - \frac{\Delta^{p\prime}}{\lambda_c^{p\prime}} \right] (1 - s^p f^p) + \frac{1}{\mathtt{u}'(c_e^p)} \left[\mathbb{E}_a \beta \Delta^{p\prime} - \overline{h}\right] + \frac{1}{\mathtt{u}'(c_e^p)} \psi_s \log(1-s^p) \end{aligned}$$

Simplifying

$$\begin{aligned} c_e^p e^p + (1-e^p) c_u^p \quad = & \quad e^p \left[\exp\{a\} - \mu_\epsilon\right] + \mathbf{B}_{FF}(u_t^p) - \tau_F - \kappa_v (1-e^p) s^p \theta^p - e^p \psi_\epsilon \log(1-\xi^p) \\ & +e^p \xi^p \left(\mathbb{E}_a \beta \frac{\lambda_c^{p\prime}}{\lambda_c^p} \left[\frac{\frac{\partial \mathtt{W}}{\partial e^{p\prime}}}{\lambda_c^{p\prime}} - \frac{\Delta^{p\prime}}{\lambda_c^{p\prime}} \right] (1 - s^p f^p) + \frac{1}{\mathtt{u}'(c_e^p)} \left[\mathbb{E}_a \beta \Delta^{p\prime} - \overline{h}\right] \right. \\ & \left. + \frac{1}{\mathtt{u}'(c_e^p)} \psi_s \log(1-s^p) \right) \end{aligned}$$

Dividing by employment:

$$\begin{aligned} c_e^p + \frac{1-e^p}{e^p}\left[c_u^p + \kappa_v s^p \theta^p\right] &= \left[\exp\{a\} - \mu_\epsilon\right] + \frac{\mathbf{B}_F(u_t^p) - \tau_F}{e^p} - \psi_\epsilon \log(1-\xi^p) \\ &\quad + \xi^p \left(\mathbb{E}_a \beta \frac{\lambda_c^{p\prime}}{\lambda_c^p} \left[\frac{\frac{\partial \mathtt{W}}{\partial e^{p\prime}}}{\lambda_c^{p\prime}} - \frac{\Delta^{p\prime}}{\lambda_c^{p\prime}} \right] (1 - s^p f^p) + \frac{1}{\mathtt{u}'(c_e^p)} \left[\mathbb{E}_a \beta \Delta^{p\prime} - \overline{h}\right] \right. \\ &\quad \left. + \frac{1}{\mathtt{u}'(c_e^p)} \psi_s \log(1 - s^p) \right) \end{aligned}$$

Rearranging

$$\begin{aligned} &\frac{1}{e^p} c_u^p + \frac{1-e^p}{e^p} \kappa_v s^p \theta^p - \frac{\mathbf{B}_F(u_t^p) - \tau_F}{e^p} \\ &- \xi^p \left(\mathbb{E}_a \beta \frac{\lambda_c^{p\prime}}{\lambda_c^p} \left[\frac{\frac{\partial \mathtt{W}}{\partial e^{p\prime}}}{\lambda_c^{p\prime}} - \frac{\Delta^{p\prime}}{\lambda_c^{p\prime}} \right] (1 - s^p f^p) + \frac{1}{\mathtt{u}'(c_e^p)} \left[\mathbb{E}_a \beta \Delta^{p\prime} - \overline{h}\right] \right. \\ &\quad \left. + \frac{1}{\mathtt{u}'(c_e^p)} \psi_s \log(1 - s^p) \right) \\ &= \left[\exp\{a\} - \mu_\epsilon\right] - c_e^p + c_e^p - \psi_\epsilon \log(1 - \xi^p) \end{aligned} \tag{55}$$

$$J^p = \frac{c_u^p - \mathbf{B}_F(u_t^p) + \tau_f - e^p \mathbf{B}_F'(u_t^p) + \kappa_v s^p \theta^p}{e^p} + E_a \left[\beta \frac{\lambda_c^{p\prime}}{\lambda_c^p} J^{p\prime} \right] (1 - s^p f^p)(1 - \xi^p). \tag{56}$$

A.2.3 The ratio of marginal utilities next period is measurable this period

This section shows that the planner promises marginal utilities of consumption in the next period such that the ratio of these, $\mathtt{u}'(c_u')/(\mathtt{u}'(c_e'))$ is measurable in this period, that is not contingent on next period's state of the economy beyond of what is already known today. As a special case, for CRRA utility, under the planner's allocations, the "consumption-based replacement rate" next period, $c_u^{p\prime}/c_e^{p\prime}$, therefore, is known already in this period.
To see that, first observe that

$$\frac{\partial s^p}{\partial \Delta^{p\prime}} = s^p (1 - s^p) \frac{\beta}{\psi_s} f^p.$$

The promise-keeping constraint, equation (46), was

$$\begin{aligned}
\beta\frac{\lambda_\Delta^p}{\lambda_c^p}(1-\xi^p) &= \beta\frac{\lambda_c^{p\prime}}{\lambda_c^p}\frac{\frac{\partial \mathtt{W}}{\partial \Delta^{p\prime}}}{\lambda_c^{p\prime}} + \frac{\lambda_\Delta^p}{\lambda_c^p}\frac{\psi_s}{1-s^p}(1-\xi^p)\frac{\partial s^p}{\partial \Delta^{p\prime}} \\
&\quad -[1-(1-\xi^p)e^p]f^p\beta\mathbb{E}_a\left\{\frac{\Delta^{p\prime}}{\lambda_c^p}\right\}\frac{\partial s^p}{\partial \Delta^{p\prime}} \\
&\quad +\mathbb{E}_a\beta\frac{\lambda_c^{p\prime}}{\lambda_c^p}\frac{\frac{\partial \mathtt{W}}{\partial e^{p\prime}}}{\lambda_c^{p\prime}}f^p\frac{\partial s^p}{\partial \Delta^{p\prime}}[1-(1-\xi^p)e^p] - \kappa_v\frac{\partial s^p}{\partial \Delta^{p\prime}}\theta^p[1-(1-\xi^p)e^p].
\end{aligned}$$

First observe that, by envelope condition (47),

$$\frac{\lambda_c^{p\prime}}{\lambda_c^p}\frac{\frac{\partial \mathtt{W}}{\partial \Delta^{p\prime}}}{\lambda_c^{p\prime}} = \frac{\lambda_c^{p\prime}}{\lambda_c^p}\frac{\lambda_\Delta^\prime}{\lambda_c^{p\prime}} = \frac{\lambda_\Delta^\prime}{\lambda_c^p}.$$

Then observe that $\partial s^p/(\partial\Delta^{p\prime})$ is measurable this period. As a result, all terms in the promise-keeping constraint apart from $\lambda_\Delta^\prime$ are measurable this period. Therefore also $\lambda_\Delta^\prime$ needs to be measurable this period. $\lambda_\Delta^\prime$ therefore is independent of the realization of the future shock.

Now, use the first-order conditions for consumption which imply equation (40). The latter equation is repeated here for convenience:

$$\lambda_\Delta^p = \frac{[\mathtt{u}'(c_e^p)-\mathtt{u}'(c_u^p)]e^p(1-e^p)}{\mathtt{u}'(c_e^p)(1-e^p)+e^p\mathtt{u}'(c_u^p)}$$

Rearranging this, and moving one period forward,

$$\lambda_\Delta^{p\prime} = \frac{\left[1-\frac{\mathtt{u}'(c_u^{p\prime})}{\mathtt{u}'(c_e^{p\prime})}\right]e^{p\prime}(1-e^{p\prime})}{(1-e^{p\prime})+e^{p\prime}\frac{\mathtt{u}'(c_u^{p\prime})}{\mathtt{u}'(c_e^{p\prime})}}.$$

Employment at the *beginning* of next period is known as of this period, compare employment-flow equation (39). Therefore, if $\lambda_\Delta^{p\prime}$ is measurable this period, so needs to be the ratio of next period's marginal utilities $\frac{\mathtt{u}'(c_u^{p\prime})}{\mathtt{u}'(c_e^{p\prime})}$. The claim regarding the replacement rate follows from the fact that for CRRA utility

$$\frac{\mathtt{u}'(c_u^{p\prime})}{\mathtt{u}'(c_e^{p\prime})} = \left(\frac{c_u^{p\prime}}{c_e^{p\prime}}\right)^{-\sigma},$$

where $\sigma > 0$ is the coefficient of relative risk aversion.

A.2.4 Derivation of planner's "free entry" and "bargaining" equations

As a preparation for the decentralization of the allocation, we derive two further intermediate results. First, we derive a planner counterpart to the vacancy posting free entry condition in the decentralized economy. Then, we derive a counterpart to the bargaining first-order condition for the planner. In the following derivations, we use two equations, the first-order condition for hiring, equation (45), and the transformed promised-utility first-order condition, equation (46).

Derivation of the "planner's bargaining equation"

We start by deriving the planner's equivalent of a "bargaining equation."
First, divide both sides of (45) by $\frac{1}{s^p}\frac{\partial s^p}{\partial \theta^p}$. This gives:

$$\begin{aligned} 0 &= -\left[\frac{\kappa_v s^p}{\frac{\partial s^p}{\partial \theta^p}} + \kappa_v \theta^p\right] - \mathbb{E}_a \beta \frac{\Delta^{p\prime}}{\lambda_c^p} f^p + \mathbb{E}_a \beta \frac{\lambda_c^{p\prime}}{\lambda_c^p} \frac{\frac{\partial \mathtt{W}}{\partial e^{p\prime}}}{\lambda_c^{p\prime}} \left[\frac{\gamma \frac{f^p}{\theta^p} s^p}{\frac{\partial s^p}{\partial \theta^p}} + f^p\right] \\ &\quad + \frac{\lambda_\Delta^p}{\lambda_c^p} \psi_s \frac{1}{1-s^p} \frac{1-\xi^p}{[\xi^p e^p + (1-e^p)]}. \end{aligned}$$

Rearrange to get

$$\begin{aligned} 0 &= -\left[\frac{\kappa_v s^p}{\frac{\partial s^p}{\partial \theta^p}} + \kappa_v \theta^p\right] + \mathbb{E}_a \beta \frac{\lambda_c^{p\prime}}{\lambda_c^p} \frac{\frac{\partial \mathtt{W}}{\partial e^{p\prime}}}{\lambda_c^{p\prime}} \left[\frac{\gamma \frac{f^p}{\theta^p} s^p}{\frac{\partial s^p}{\partial \theta^p}}\right] + f^p \mathbb{E}_a \beta \frac{\lambda_c^{p\prime}}{\lambda_c^p} \left[\frac{\frac{\partial \mathtt{W}}{\partial e^{p\prime}}}{\lambda_c^{p\prime}} - \frac{\Delta^{p\prime}}{\lambda_c^p}\right] \\ &\quad + \frac{\lambda_\Delta^p}{\lambda_c^p} \psi_s \frac{1}{1-s^p} \frac{1-\xi^p}{[\xi^p e^p + (1-e^p)]}. \end{aligned}$$

Use our definition $J^p := \frac{\frac{\partial \mathtt{W}}{\partial e^p}}{\lambda_c^p} - \frac{\Delta^p}{\lambda_c^p}$,introduced before equation (53), and rearrange further:

$$-\kappa_v \theta^p + \mathbb{E}_a \beta \frac{\lambda_c^{p\prime}}{\lambda_c^p} J^{p\prime} f^p = \frac{\kappa_v s^p}{\frac{\partial s^p}{\partial \theta^p}} - \mathbb{E}_a \beta \frac{\lambda_c^{p\prime}}{\lambda_c^p} \frac{\frac{\partial \mathtt{W}}{\partial e^{p\prime}}}{\lambda_c^{p\prime}} \frac{\gamma \frac{f^p}{\theta^p} s^p}{\frac{\partial s^p}{\partial \theta^p}} - \frac{\lambda_\Delta^p}{\lambda_c^p} \psi_s \frac{1}{1-s^p} \frac{1-\xi^p}{[\xi^p e^p + (1-e^p)]}. \quad (57)$$

The next steps use the promised-utility equation (46). Weight promised-utility equation (46) on each side by the density of the future state and integrate over all possible states – that is, take the expectation as of this period of both left-hand side and right-hand side. This gives

$$\begin{aligned} \beta \frac{\lambda_\Delta^p}{\lambda_c^p}(1-\xi^p) &= \beta \mathbb{E}_a \frac{\lambda_c^{p\prime}}{\lambda_c^p} \frac{\frac{\partial \mathtt{W}}{\partial \Delta^{p\prime}}}{\lambda_c^{p\prime}} + \frac{\lambda_\Delta^p}{\lambda_c^p} \frac{\psi_s}{1-s^p} (1-\xi^p) \frac{\partial s^p}{\partial \Delta^{p\prime}} \\ &\quad -[1-(1-\xi^p)e^p] f^p \beta \mathbb{E}_a \frac{\Delta^{p\prime}}{\lambda_c^p} \frac{\partial s^p}{\partial \Delta^{p\prime}} \\ &\quad + \mathbb{E}_a \beta \frac{\lambda_c^{p\prime}}{\lambda_c^p} \frac{\frac{\partial \mathtt{W}}{\partial e^{p\prime}}}{\lambda_c^{p\prime}} f^p \frac{\partial s^p}{\partial \Delta^{p\prime}} [1-(1-\xi^p)e^p] - \kappa_v \frac{\partial s^p}{\partial \Delta^{p\prime}} \theta^p [1-(1-\xi^p)e^p]. \end{aligned}$$

Divide both sides by $\frac{\partial s^p}{\partial \Delta^{p\prime}}$ and rearrange slightly to obtain

$$\begin{aligned}\frac{\beta\frac{\lambda_\Delta^p}{\lambda_c^p}(1-\xi^p)-\beta\mathbb{E}_a\frac{\lambda_c^{p\prime}}{\lambda_c^p}\frac{\frac{\partial \mathtt{W}}{\partial \Delta^{p\prime}}}{\lambda_c^{p\prime}}}{\frac{\partial s^p}{\partial \Delta^{p\prime}}} &= \frac{\lambda_\Delta^p}{\lambda_c^p}\frac{\psi_s}{1-s^p}(1-\xi^p)-[1-(1-\xi^p)e^p]f^p\beta\mathbb{E}_a\frac{\Delta^{p\prime}}{\lambda_c^p} \\ &+[1-(1-\xi^p)e^p]f^p\mathbb{E}_a\beta\frac{\lambda_c^{p\prime}}{\lambda_c^p}\frac{\frac{\partial \mathtt{W}}{\partial e^{p\prime}}}{\lambda_c^{p\prime}}-\kappa_v\theta^p[1-(1-\xi^p)e^p].\end{aligned}$$

Rearrange further to get

$$\begin{aligned}\frac{\beta\frac{\lambda_\Delta^p}{\lambda_c^p}(1-\xi^p)-\beta\mathbb{E}_a\frac{\lambda_c^{p\prime}}{\lambda_c^p}\frac{\frac{\partial \mathtt{W}}{\partial \Delta^{p\prime}}}{\lambda_c^{p\prime}}}{\frac{\partial s^p}{\partial \Delta^{p\prime}}} &= \frac{\lambda_\Delta^p}{\lambda_c^p}\frac{\psi_s}{1-s^p}(1-\xi^p) \\ &+[1-(1-\xi^p)e^p]f^p\mathbb{E}_a\beta\frac{\lambda_c^{p\prime}}{\lambda_c^p}\left[\frac{\frac{\partial \mathtt{W}}{\partial e^{p\prime}}}{\lambda_c^{p\prime}}-\frac{\Delta^{p\prime}}{\lambda_c^{p\prime}}\right] \\ &-\kappa_v\theta^p[1-(1-\xi^p)e^p].\end{aligned}$$

Use again the definition of $J^{p\prime}=\frac{\frac{\partial \mathtt{W}}{\partial e^{p\prime}}}{\lambda_c^{p\prime}}-\frac{\Delta^{p\prime}}{\lambda_c^{p\prime}}$ to simplify the expression to

$$\frac{\beta\frac{\lambda_\Delta^p}{\lambda_c^p}(1-\xi^p)-\beta\mathbb{E}_a\frac{\lambda_c^{p\prime}}{\lambda_c^p}\frac{\frac{\partial \mathtt{W}}{\partial \Delta^{p\prime}}}{\lambda_c^{p\prime}}}{\frac{\partial s^p}{\partial \Delta^{p\prime}}}=[\xi^pe^p+(1-e^p)]f^p\,\mathbb{E}_a\beta\frac{\lambda_c^{p\prime}}{\lambda_c^p}J^{p\prime}-\kappa_v\theta^p[1-(1-\xi^p)e^p]+\frac{\lambda_\Delta^p}{\lambda_c^p}\frac{\psi_s}{1-s^p}(1-\xi^p).$$

Next, use that the participation constraint, equation (37), implies $\frac{\partial s^p}{\partial \Delta^{p\prime}}=s^p(1-s^p)\frac{\beta}{\psi_s}f^p$:

$$\frac{\beta\frac{\lambda_\Delta^p}{\lambda_c^p}(1-\xi^p)-\beta\mathbb{E}_a\frac{\lambda_c^{p\prime}}{\lambda_c^p}\frac{\frac{\partial \mathtt{W}}{\partial \Delta^{p\prime}}}{\lambda_c^{p\prime}}}{s^p(1-s^p)\frac{\beta}{\psi_s}f^p}=[\xi^pe^p+(1-e^p)]f^p\,\mathbb{E}_a\beta\frac{\lambda_c^{p\prime}}{\lambda_c^p}J^{p\prime}-\kappa_v\theta^p[1-(1-\xi^p)e^p]+\frac{\lambda_\Delta^p}{\lambda_c^p}\frac{\psi_s}{1-s^p}(1-\xi^p).$$

Divide both sides by $[\xi^pe^p+(1-e^p)]$.

$$\frac{\beta\frac{\lambda_\Delta^p}{\lambda_c^p}(1-\xi^p)-\beta\mathbb{E}_a\frac{\lambda_c^{p\prime}}{\lambda_c^p}\frac{\frac{\partial \mathtt{W}}{\partial \Delta^{p\prime}}}{\lambda_c^{p\prime}}}{s^p(1-s^p)\frac{\beta}{\psi_s}f^p[\xi^pe^p+(1-e^p)]}=f^p\,\mathbb{E}_a\beta\frac{\lambda_c^{p\prime}}{\lambda_c^p}J^{p\prime}-\kappa_v\theta^p+\frac{\lambda_\Delta^p}{\lambda_c^p}\frac{\frac{\psi_s}{1-s^p}(1-\xi^p)}{[\xi^pe^p+(1-e^p)]}.$$

Rearrange to get:

$$-\kappa_v\theta^p+\mathbb{E}_a\beta\frac{\lambda_c^{p\prime}}{\lambda_c^p}J^{p\prime}f^p=\frac{\beta\frac{\lambda_\Delta^p}{\lambda_c^p}(1-\xi^p)-\beta\mathbb{E}_a\frac{\lambda_c^{p\prime}}{\lambda_c^p}\frac{\frac{\partial \mathtt{W}}{\partial \Delta^{p\prime}}}{\lambda_c^{p\prime}}}{s^p(1-s^p)\frac{\beta}{\psi_s}f^p[\xi^pe^p+(1-e^p)]}-\frac{\lambda_\Delta^p}{\lambda_c^p}\frac{\frac{\psi_s}{1-s^p}(1-\xi^p)}{[\xi^pe^p+(1-e^p)]}. \tag{58}$$

Next, observe that the left-hand sides of equations (57), derived from the planner's hiring first-order condition, and (58), derived from the planner's promised-utility first-order condition, are equal. Equating, therefore, the right-hand sides of these two equations gives:

$$\frac{\kappa_v s^p}{\frac{\partial s^p}{\partial \theta^p}} - \mathbb{E}_a \beta \frac{\lambda_c^{p\prime}}{\lambda_c^p} \frac{\frac{\partial \mathtt{W}}{\partial e^{p\prime}}}{\lambda_c^{p\prime}} \frac{\gamma \frac{f^p}{\theta^p} s^p}{\frac{\partial s^p}{\partial \theta^p}} - \frac{\lambda_\Delta^p}{\lambda_c^p} \frac{\psi_s}{1-s^p} \frac{1-\xi^p}{[\xi^p e^p + (1-e^p)]}$$

$$= \frac{\beta \frac{\lambda_\Delta^p}{\lambda_c^p}(1-\xi^p) - \beta \mathbb{E}_a \frac{\lambda_c^{p\prime}}{\lambda_c^p} \frac{\frac{\partial \mathtt{W}}{\partial \Delta^{p\prime}}}{\lambda_c^{p\prime}}}{s^p(1-s^p)\frac{\beta}{\psi_s} f^p [\xi^p e^p + (1-e^p)]} - \frac{\lambda_\Delta^p}{\lambda_c^p} \frac{\psi_s}{1-s^p} \frac{1-\xi^p}{[\xi^p e^p + (1-e^p)]}.$$

Next, note that the final terms on each side of the equation cancel, so:

$$\frac{\kappa_v s^p}{\frac{\partial s^p}{\partial \theta^p}} - \mathbb{E}_a \beta \frac{\lambda_c^{p\prime}}{\lambda_c^p} \frac{\frac{\partial \mathtt{W}}{\partial e^{p\prime}}}{\lambda_c^{p\prime}} \frac{\gamma \frac{f^p}{\theta^p} s^p}{\frac{\partial s^p}{\partial \theta^p}} \quad = \quad \frac{\beta \frac{\lambda_\Delta^p}{\lambda_c^p}(1-\xi^p) - \beta \mathbb{E}_a \frac{\lambda_c^{p\prime}}{\lambda_c^p} \frac{\frac{\partial \mathtt{W}}{\partial \Delta^{p\prime}}}{\lambda_c^{p\prime}}}{s^p(1-s^p)\frac{\beta}{\psi} f^p [\xi^p e^p + (1-e^p)]}.$$

Multiply through by $\frac{\partial s^p}{\partial \theta^p}$:

$$\kappa_v s^p - \mathbb{E}_a \beta \frac{\lambda_c^{p\prime}}{\lambda_c^p} \frac{\frac{\partial \mathtt{W}}{\partial e^{p\prime}}}{\lambda_c^{p\prime}} \gamma \frac{f^p}{\theta^p} s^p \quad = \quad \frac{\beta \frac{\lambda_\Delta^p}{\lambda_c^p}(1-\xi^p) - \beta \mathbb{E}_a \frac{\lambda_c^{p\prime}}{\lambda_c^p} \frac{\frac{\partial \mathtt{W}}{\partial \Delta^{p\prime}}}{\lambda_c^{p\prime}}}{s^p(1-s^p)\frac{\beta}{\psi} f^p [\xi^p e^p + (1-e^p)]} \frac{\partial s^p}{\partial \theta^p}.$$

Next use, from the participation constraint (37), that $\frac{\partial s^p}{\partial \theta^p} = s^p(1-s^p)\frac{\beta}{\psi_s}\mathbb{E}_a \Delta^{p\prime} \gamma \frac{f^p}{\theta^p}$ to obtain

$$\kappa_v s^p - \mathbb{E}_a \beta \frac{\lambda_c^{p\prime}}{\lambda_c^p} \frac{\frac{\partial \mathtt{W}}{\partial e^{p\prime}}}{\lambda_c^{p\prime}} \gamma \frac{f^p}{\theta^p} s^p \quad = \quad \frac{\beta \frac{\lambda_\Delta^p}{\lambda_c^p}(1-\xi^p) - \beta \mathbb{E}_a \frac{\lambda_c^{p\prime}}{\lambda_c^p} \frac{\frac{\partial \mathtt{W}}{\partial \Delta^{p\prime}}}{\lambda_c^{p\prime}}}{s^p(1-s^p)\frac{\beta}{\psi_s} f^p [\xi^p e^p + (1-e^p)]} s^p(1-s^p)\frac{\beta}{\psi_s}\mathbb{E}_a \Delta^{p\prime} \gamma \frac{f^p}{\theta^p}.$$

Divide both sides by $s^p \gamma \frac{f^p}{\theta^p}$, and cancel $(1-s)$ and $\frac{\beta}{\psi_s}$ in the denominator and numerator of the right-hand side:

$$\kappa_v \frac{\theta^p}{\gamma f^p} - \mathbb{E}_a \beta \frac{\lambda_c^{p\prime}}{\lambda_c^p} \frac{\frac{\partial \mathtt{W}}{\partial e^{p\prime}}}{\lambda_c^{p\prime}} \quad = \quad \frac{\beta \frac{\lambda_\Delta^p}{\lambda_c^p}(1-\xi^p) - \beta \mathbb{E}_a \frac{\lambda_c^{p\prime}}{\lambda_c^p} \frac{\frac{\partial \mathtt{W}}{\partial \Delta^{p\prime}}}{\lambda_c^{p\prime}}}{s^p f^p [\xi^p e^p + (1-e^p)]} \mathbb{E}_a \Delta^{p\prime}.$$

Collecting terms we obtain:

$$\kappa_v \frac{\theta^p}{\gamma f^p} - \mathbb{E}_a \beta \frac{\lambda_c^{p\prime}}{\lambda_c^p} \frac{\frac{\partial \mathtt{W}}{\partial e^{p\prime}}}{\lambda_c^{p\prime}} \quad = \quad \frac{\frac{\lambda_\Delta^p}{\lambda_c^p}(1-\xi^p)\beta \mathbb{E}_a \Delta^{p\prime}}{s^p f^p [\xi^p e^p + (1-e^p)]} - \frac{\beta \mathbb{E}_a \frac{\lambda_c^{p\prime}}{\lambda_c^p} \frac{\frac{\partial \mathtt{W}}{\partial \Delta^{p\prime}}}{\lambda_c^{p\prime}}}{s^p f^p [\xi^p e^p + (1-e^p)]} \mathbb{E}_a \Delta^{p\prime}.$$

Next, observe that, by envelope condition (47), $\frac{\frac{\partial \mathtt{W}}{\partial \Delta^{p\prime}}}{\lambda_c^{p\prime}} = \frac{\lambda_\Delta^{p\prime}}{\lambda_c^{p\prime}}$. Using this we obtain:

$$\kappa_v \frac{\theta^p}{\gamma f^p} - \mathbb{E}_a \beta \frac{\lambda_c^{p\prime}}{\lambda_c^p} \frac{\frac{\partial \mathtt{W}}{\partial e^{p\prime}}}{\lambda_c^{p\prime}} \quad = \quad \frac{\frac{\lambda_\Delta^p}{\lambda_c^p}(1-\xi^p)\beta \mathbb{E}_a \Delta^{p\prime}}{s^p f^p [\xi^p e^p + (1-e^p)]} - \frac{\beta \mathbb{E}_a \frac{\lambda_c^{p\prime}}{\lambda_c^p} \frac{\lambda_\Delta^{p\prime}}{\lambda_c^{p\prime}}}{s^p f^p [\xi^p e^p + (1-e^p)]} \mathbb{E}_a \Delta^{p\prime}.$$

Bring the last term on the left-hand side to the right-hand side, and expand the right-hand side by adding and substracting $\beta\mathbb{E}_a\frac{\lambda_c^{p\prime}}{\lambda_c^p}\frac{\Delta^{p\prime}}{\lambda_c^{p\prime}}$:

$$\kappa_v\frac{\theta^p}{\gamma f^p} = \mathbb{E}_a\beta\frac{\lambda_c^{p\prime}}{\lambda_c^p}\left[\frac{\frac{\partial \mathtt{W}}{\partial e^{p\prime}}}{\lambda_c^{p\prime}} - \frac{\Delta^{p\prime}}{\lambda_c^{p\prime}}\right] + \beta\mathbb{E}_a\frac{\lambda_c^{p\prime}}{\lambda_c^p}\frac{\Delta^{p\prime}}{\lambda_c^{p\prime}} + \frac{\frac{\lambda_\Delta^p}{\lambda_c^p}(1-\xi^p)\beta\mathbb{E}_a\Delta^{p\prime}}{s^p f^p[\xi^p e^p + (1-e^p)]} - \frac{\beta\mathbb{E}_a\frac{\lambda_c^{p\prime}}{\lambda_c^p}\frac{\lambda_\Delta^{p\prime}}{\lambda_c^{p\prime}}}{s^p f^p[\xi^p e^p + (1-e^p)]}\mathbb{E}_a\Delta^{p\prime}.$$

The term in square brackets is the definition of $J^{p\prime}$ that we have used repeatedly before. Using this, we have:

$$\kappa_v\frac{\theta^p}{\gamma f^p} = \mathbb{E}_a\beta\frac{\lambda_c^{p\prime}}{\lambda_c^p}J^{p\prime} + \beta\mathbb{E}_a\frac{\lambda_c^{p\prime}}{\lambda_c^p}\frac{\Delta^{p\prime}}{\lambda_c^{p\prime}} + \frac{\frac{\lambda_\Delta^p}{\lambda_c^p}(1-\xi^p)\beta\mathbb{E}_a\Delta^{p\prime}}{s^p f^p[\xi^p e^p + (1-e^p)]} - \frac{\beta\mathbb{E}_a\frac{\lambda_c^{p\prime}}{\lambda_c^p}\frac{\lambda_\Delta^{p\prime}}{\lambda_c^{p\prime}}}{s^p f^p[\xi^p e^p + (1-e^p)]}\mathbb{E}_a\Delta^{p\prime}.$$

Rearrange to get

$$\kappa_v\frac{\theta^p}{\gamma f^p} = \beta\mathbb{E}_a\frac{\lambda_c^{p\prime}}{\lambda_c^p}J^{p\prime} + \beta\mathbb{E}_a\frac{\lambda_c^{p\prime}}{\lambda_c^p}\frac{\Delta^{p\prime}}{\lambda_c^{p\prime}} + \frac{\beta\mathbb{E}_a\frac{\Delta^{p\prime}}{\lambda_c^p}[\lambda_\Delta^p(1-\xi^p) - \mathbb{E}_a\lambda_\Delta^{p\prime}]}{[\xi^p e^p + (1-e^p)]f^p s^p}$$

Next, use the planner's first-order conditions for consumption when employed or unemployed, equations (40) and (41). These imply that $e^p\left(1-\frac{\lambda_c^p}{\mathtt{u}'(c_e^p)}\right) = \frac{[\mathtt{u}'(c_e^p)-\mathtt{u}'(c_u^p)](1-e^p)e^p}{\mathtt{u}'(c_e^p)(1-e^p)+e^p\,\mathtt{u}'(c_u^p)} = \lambda_\Delta^p$. Use this to substitute for λ_Δ^p and $\lambda_\Delta^{p\prime}$ to get

$$\kappa_v\frac{\theta^p}{\gamma f^p} = \beta\mathbb{E}_a\frac{\lambda_c^{p\prime}}{\lambda_c^p}J^{p\prime} + \beta\mathbb{E}_a\frac{\lambda_c^{p\prime}}{\lambda_c^p}\frac{\Delta^{p\prime}}{\lambda_c^{p\prime}} + \frac{\beta\mathbb{E}_a\frac{\Delta^{p\prime}}{\lambda_c^p}\left[e^p\left(1-\frac{\lambda_c^p}{\mathtt{u}'(c_e^p)}\right)(1-\xi^p) - \mathbb{E}_a e^{p\prime}\left(1-\frac{\lambda_c^{p\prime}}{\mathtt{u}'(c_e^{p\prime})}\right)\right]}{[\xi^p e^p + (1-e^p)]f^p s^p}.$$

Next, use the law of motion for employment, equation (39), which reads

$$e^{p\prime} = e^p(1-\xi^p) + [\xi^p e^p + (1-e^p)]s^p f^p,$$

to rewrite the above as

$$\begin{aligned}
\kappa_v\frac{\theta^p}{\gamma f^p} &= \beta\mathbb{E}_a\frac{\lambda_c^{p\prime}}{\lambda_c^p}J^{p\prime} + \beta\mathbb{E}_a\frac{\lambda_c^{p\prime}}{\lambda_c^p}\frac{\Delta^{p\prime}}{\lambda_c^{p\prime}} + \frac{\beta\mathbb{E}_a\frac{\Delta^{p\prime}}{\lambda_c^p}\left[e^p(1-\xi^p) - e^{p\prime} - \frac{\lambda_c^p}{\mathtt{u}'(c_e^p)}e^p(1-\xi^p) + \mathbb{E}_a e^{p\prime}\frac{\lambda_c^{p\prime}}{\mathtt{u}'(c_e^{p\prime})}\right]}{[\xi^p e^p + (1-e^p)]f^p s^p} \\
&= \beta\mathbb{E}_a\frac{\lambda_c^{p\prime}}{\lambda_c^p}J^{p\prime} + \beta\mathbb{E}_a\frac{\lambda_c^{p\prime}}{\lambda_c^p}\frac{\Delta^{p\prime}}{\lambda_c^{p\prime}} + \frac{\beta\mathbb{E}_a\frac{\Delta^{p\prime}}{\lambda_c^p}\left[-[\xi^p e^p + (1-e^p)]s^p f - \frac{\lambda_c^p}{\mathtt{u}'(c_e^p)}e^p(1-\xi^p) + \mathbb{E}_a e^{p\prime}\frac{\lambda_c^{p\prime}}{\mathtt{u}'(c_e^{p\prime})}\right]}{[\xi^p e^p + (1-e^p)]f^p s^p} \\
&= \beta\mathbb{E}_a\frac{\lambda_c^{p\prime}}{\lambda_c^p}J^{p\prime} + \beta\mathbb{E}_a\frac{\lambda_c^{p\prime}}{\lambda_c^p}\frac{\Delta^{p\prime}}{\lambda_c^{p\prime}} - \beta\mathbb{E}_a\frac{\lambda_c^{p\prime}}{\lambda_c}\frac{\Delta^{p\prime}}{\lambda_c^{p\prime}} + \frac{\beta\mathbb{E}_a\frac{\Delta^{p\prime}}{\lambda_c^p}\left[-\frac{\lambda_c^p}{\mathtt{u}'(c_e^p)}e^p(1-\xi^p) + \mathbb{E}_a e^{p\prime}\frac{\lambda_c^{p\prime}}{\mathtt{u}'(c_e^{p\prime})}\right]}{[\xi^p e^p + (1-e^p)]f^p s^p} \\
&= \beta\mathbb{E}_a\frac{\lambda_c^{p\prime}}{\lambda_c^p}J^{p\prime} + \frac{\beta\mathbb{E}_a\frac{\Delta^{p\prime}}{\lambda_c^p}\left[-\frac{\lambda_c^p}{\mathtt{u}'(c_e^p)}e^p(1-\xi^p) + \mathbb{E}_a e^{p\prime}\frac{\lambda_c^{p\prime}}{\mathtt{u}'(c_e^{p\prime})}\right]}{[\xi^p e^p + (1-e^p)]f^p s^p}.
\end{aligned}$$

Using again the law of motion for employment (see above), we obtain

$$\kappa_v \frac{\theta^p}{\gamma f^p} = \beta \mathbb{E}_a \frac{\lambda_c^{p\prime}}{\lambda_c^p} J^{p\prime} + \beta \mathbb{E}_a \frac{\lambda_c^{p\prime}}{\lambda_c^p} \frac{\Delta^{p\prime}}{\lambda_c^{p\prime}} \mathbb{E}_a \frac{\lambda_c^{p\prime}}{\mathrm{u}'(c_e^{p\prime})} + \frac{-e^p(1-\xi^p)\beta \mathbb{E}_a \frac{\Delta^{p\prime}}{\lambda_c^p} \frac{\lambda_c^p}{\mathrm{u}'(c_e^p)} + e^p(1-\xi^p)\beta \mathbb{E}_a \frac{\Delta^{p\prime}}{\lambda_c^p} \mathbb{E}_a \frac{\lambda_c^{p\prime}}{\mathrm{u}'(c_e^{p\prime})}}{[\xi^p e^p + (1-e^p)] f^p s^p}.$$

By the planner's first-order conditions for consumption we have that $\frac{\lambda_c^{p\prime}}{\mathrm{u}'(c_e^{p\prime})} = \frac{1}{\frac{\mathrm{u}'\left(c_e^{p\prime}\right)}{\mathrm{u}'\left(c_u^{p\prime}\right)}(1-e^{p\prime})+e^{p\prime}}$, compare equation (41). In addition, notice that this is measurable with respect to the current period's information set; compare the employment flow equation (39) and the discussion in Section A.2.3. We therefore have that

$$\begin{aligned}
\kappa_v \frac{\theta^p}{\gamma f^p} \quad = \quad & \beta \mathbb{E}_a \frac{\lambda_c^{p\prime}}{\lambda_c^p} J^{p\prime} + \left[\frac{1}{\frac{\mathrm{u}'\left(c_e^{p\prime}\right)}{\mathrm{u}'\left(c_u^{p\prime}\right)}(1-e^{p\prime})+e^{p\prime}}\right] \beta \mathbb{E}_a \frac{\lambda_c^{p\prime}}{\lambda_c^p} \frac{\Delta^{p\prime}}{\lambda_c^{p\prime}} \\
& + e^p(1-\xi^p)\beta \mathbb{E}_a \frac{\Delta^{p\prime}}{\lambda_c^p} \frac{\left[\frac{1}{\frac{\mathrm{u}'\left(c_e^{p\prime}\right)}{\mathrm{u}'\left(c_u^{p\prime}\right)}(1-e^{p\prime})+e^{p\prime}}\right] - \left[\frac{1}{\frac{\mathrm{u}'(c_e^p)}{\mathrm{u}'(c_u^p)}(1-e^p)+e^p}\right]}{[\xi^p e^p + (1-e^p)] f^p s^p}.
\end{aligned}$$

Next, observe that $\frac{1}{\frac{\mathrm{u}'\left(c_e^{p\prime}\right)}{\mathrm{u}'\left(c_u^{p\prime}\right)}(1-e^{p\prime})+e^{p\prime}} \mathbb{E}_a \frac{\lambda_c^{p\prime}}{\lambda_c^p} \frac{\Delta^{p\prime}}{\lambda_c^{p\prime}} = \mathbb{E}_a \frac{\lambda_c^{p\prime}}{\lambda_c^p} \frac{\Delta^{p\prime}}{\mathrm{u}'(c_e^{p\prime})} \underbrace{\frac{\mathrm{u}'(c_e^{p\prime})}{\lambda_c^{p\prime}} \frac{1}{\frac{\mathrm{u}'(c_e^{p\prime})}{\mathrm{u}'(c_u^{p\prime})}(1-e^{p\prime})+e^{p\prime}}}_{=1 \text{ by equation (41)}} =$
$\mathbb{E}_a \frac{\lambda_c^{p\prime}}{\lambda_c^p} \frac{\Delta^{p\prime}}{\mathrm{u}'(c_e^{p\prime})}$.
We therefore arrive at:

$$\begin{aligned}
\kappa_v \frac{\theta^p}{\gamma f^p} \quad = \quad & \beta \mathbb{E}_a \frac{\lambda_c^{p\prime}}{\lambda_c^p} J^{p\prime} + \beta \mathbb{E}_a \frac{\lambda_c^{p\prime}}{\lambda_c^p} \frac{\Delta^{p\prime}}{\mathrm{u}'(c_e^{p\prime})} \\
& + e^p(1-\xi^p)\beta \mathbb{E}_a \frac{\Delta^{p\prime}}{\lambda_c^p} \frac{\left[\frac{1}{\frac{\mathrm{u}'\left(c_e^{p\prime}\right)}{\mathrm{u}'\left(c_u^{p\prime}\right)}(1-e^{p\prime})+e^{p\prime}}\right] - \left[\frac{1}{\frac{\mathrm{u}'(c_e^p)}{\mathrm{u}'(c_u^p)}(1-e^p)+e^p}\right]}{[\xi^p e^p + (1-e^p)] f^p s^p}.
\end{aligned}$$

Doing that step again, we have that

$$\begin{aligned}
\kappa_v \frac{\theta^p}{\gamma f^p} \quad = \quad & \beta \mathbb{E}_a \frac{\lambda_c^{p\prime}}{\lambda_c^p} J^{p\prime} + \beta \mathbb{E}_a \frac{\lambda_c^{p\prime}}{\lambda_c^p} \frac{\Delta^{p\prime}}{\mathrm{u}'(c_e^{p\prime})} \\
& + \frac{e^p(1-\xi^p)\beta \mathbb{E}_a \frac{\lambda_c^{p\prime}}{\lambda_c^p} \frac{\Delta^{p\prime}}{\mathrm{u}'(c_e^{p\prime})} \left[1 - \frac{\frac{\mathrm{u}'(c_e^{p\prime})}{\mathrm{u}'(c_u^{p\prime})}(1-e^{p\prime})+e^{p\prime}}{\frac{\mathrm{u}'(c_e^p)}{\mathrm{u}'(c_u^p)}(1-e^p)+e^p}\right]}{[\xi^p e^p + (1-e^p)] f^p s^p}
\end{aligned}$$

Again, the term in square brackets is measurable with respect to today's information set. To neatly summarize the equation, define a wedge ς^p as

$$\varsigma^p := \frac{e^p(1-\xi^p)}{[\xi^p e^p + (1-e^p)]f^p s^p}\left[1 - \frac{\frac{\mathtt{u}'(c_e^{p\prime})}{\mathtt{u}'(c_u^{p\prime})}(1-e^{p\prime}) + e^{p\prime}}{\frac{\mathtt{u}'(c_e^{p})}{\mathtt{u}'(c_u^{p})}(1-e^{p}) + e^{p}}\right]. \tag{59}$$

Alternatively, the wedge ς can be expressed as

$$\varsigma^p = \frac{e^p(1-\xi^p)}{[\xi^p e^p + (1-e^p)]f^p s^p}\left[1 - \frac{\frac{\mathtt{u}'\left(c_e^{p\prime}\right)}{\lambda_c^{p\prime}}}{\frac{\mathtt{u}'\left(c_e^{p}\right)}{\lambda_c^{p}}}\right].$$

This shows that ς^p measures how the wedge between the planner's marginal utility of wealth and the employed workers' marginal utility evolves over time. Note that $\varsigma^p = 0$ in steady state.
With this definition of ς^p, we obtain the following "bargaining equation of the planner:"

$$\kappa_v \frac{\theta^p}{\gamma f^p} = \quad \mathbb{E}_a \beta \frac{\lambda_c^{p\prime}}{\lambda_c^p} J^{p\prime} + (1+\varsigma^p)\mathbb{E}_a \beta \frac{\lambda_c^{p\prime}}{\lambda_c^p}\frac{\Delta^{p\prime}}{\mathtt{u}'(c_e^{p\prime})}. \tag{60}$$

Derivation of the "planner's free-entry condition"

This section derives the planner's equivalent of a free-entry condition for vacancies. Use the simplified first-order condition for promised utility, equation (58), replicated here for convenience:

$$-\kappa_v\theta^p + \mathbb{E}_a\beta\frac{\lambda_c^{p\prime}}{\lambda_c^p}J^{p\prime}f^p = \frac{\beta\frac{\lambda_\Delta^p}{\lambda_c^p}(1-\xi^p) - \beta\mathbb{E}_a\frac{\lambda_c^{p\prime}}{\lambda_c^p}\frac{\frac{\partial \mathtt{W}}{\partial\Delta^{p\prime}}}{\lambda_c^{p\prime}}}{s^p(1-s^p)\frac{\beta}{\psi_s}f^p[\xi^p e^p + (1-e^p)]} - \frac{\lambda_\Delta^p}{\lambda_c^p}\frac{\frac{\psi_s}{1-s^p}(1-\xi^p)}{[\xi^p e^p + (1-e^p)]}.$$

Divide through by f^p, and rearrange the terms to obtain:

$$\begin{aligned}
\kappa_v\frac{\theta^p}{f^p} &= \mathbb{E}_a\beta\frac{\lambda_c^{p\prime}}{\lambda_c^p}J^{p\prime} - \frac{\beta\frac{\lambda_\Delta^p}{\lambda_c^p}(1-\xi^p) - \mathbb{E}_a\beta\frac{\lambda_c^{p\prime}}{\lambda_c^p}\frac{\lambda_\Delta^{p\prime}}{\lambda_c^{p\prime}}}{f^p s^p(1-s^p)\frac{\beta}{\psi_s}f^p[\xi^p e^p + (1-e^p)]} + \frac{\psi_s}{f^p(1-s^p)}\frac{\lambda_\Delta^p}{\lambda_c^p}\frac{(1-\xi^p)}{[\xi^p e^p + (1-e^p)]} \\
&= \mathbb{E}_a\beta\frac{\lambda_c^{p\prime}}{\lambda_c^p}J^{p\prime} + \frac{\psi_s}{f^p(1-s^p)}\frac{\mathbb{E}_a\frac{\lambda_c^{p\prime}}{\lambda_c^p}\frac{\lambda_\Delta^{p\prime}}{\lambda_c^{p\prime}}}{s^p f^p[\xi^p e^p + (1-e^p)]} - \frac{\psi_s}{f^p(1-s^p)}\frac{\lambda_\Delta^p}{\lambda_c^p}\frac{(1-\xi^p)(1-s^p f^p)}{s^p f^p[\xi^p e^p + (1-e^p)]} \\
&= \mathbb{E}_a\beta\frac{\lambda_c^{p\prime}}{\lambda_c^p}J^{p\prime} + \frac{\psi_s}{f^p(1-s^p)}\frac{1}{s^p f^p[\xi^p e^p + (1-e^p)]}\frac{1}{\lambda_c^p}\left[\mathbb{E}_a\lambda_\Delta^{p\prime} - \lambda_\Delta^p(1-\xi^p)(1-s^p f^p)\right].
\end{aligned}$$

Using again $\lambda_\Delta^p = e^p \left(1 - \frac{\lambda_c^p}{\mathrm{u}'(c_e^p)}\right)$, we obtain that

$$\begin{aligned}
\kappa_v \frac{\theta^p}{f^p} \quad = \quad & \mathbb{E}_a \beta \frac{\lambda_c^{p\prime}}{\lambda_c^p} J^{p\prime} \\
& + \frac{\psi_s}{f^p(1-s^p)} \frac{1}{s^p f^p[\xi^p e^p + (1-e^p)]} \\
& * \frac{1}{\lambda_c^p} \left[\mathbb{E}_a e^{p\prime} \left(1 - \frac{\lambda_c^{p\prime}}{\mathrm{u}'\left(c_e^{p\prime}\right)}\right) - e^p \left(1 - \frac{\lambda_c^p}{\mathrm{u}'\left(c_e^p\right)}\right)(1-\xi^p)(1-s^p f^p)\right].
\end{aligned}$$

Use the law of motion of employment, $e^{p\prime} = e^p(1-\xi^p)(1-s^p f^p) + s^p f^p$, to get

$$\begin{aligned}
\kappa_v \frac{\theta^p}{f^p} \quad = \quad & \mathbb{E}_a \beta \frac{\lambda_c^{p\prime}}{\lambda_c^p} J^{p\prime} \\
& + \frac{\psi_s}{f^p(1-s^p)} \frac{1}{s^p f^p[\xi^p e^p + (1-e^p)]} \frac{1}{\lambda_c^p} \\
& \cdot \left[\mathbb{E}_a e^{p\prime} \left(1 - \frac{\lambda_c^{p\prime}}{\mathrm{u}'\left(c_e^{p\prime}\right)}\right) - e^{p\prime} \left(1 - \frac{\lambda_c^p}{\mathrm{u}'(c_e^p)}\right) + s^p f \left(1 - \frac{\lambda_c^p}{\mathrm{u}'(c_e^p)}\right)\right].
\end{aligned}$$

Rearranging this:

$$\begin{aligned}
\kappa_v \frac{\theta^p}{f^p} \quad = \quad & \mathbb{E}_a \beta \frac{\lambda_c^{p\prime}}{\lambda_c^p} J^{p\prime} \\
& - \frac{\psi_s}{f^p(1-s^p)} \frac{\frac{\lambda_c^p}{\mathrm{u}'(c_e^p)} - 1}{[\xi^p e^p + (1-e^p)]} \frac{1}{\lambda_c^p} \\
& + \frac{\psi_s}{f^p(1-s^p)} \frac{1}{s^p f^p[\xi^p e^p + (1-e^p)]} \frac{1}{\lambda_c^p} \mathbb{E}_a e^{p\prime} \left(\frac{\lambda_c^p}{\mathrm{u}'(c_e^p)} - \frac{\lambda_c^{p\prime}}{\mathrm{u}'(c_e^{p\prime})}\right). \qquad (61)
\end{aligned}$$

Using again that $\frac{\lambda_c^p}{\mathrm{u}'(c_e^p)} = \frac{1}{\frac{\mathrm{u}'(c_e^p)}{\mathrm{u}'(c_u^p)}(1-e^p)+e^p}$ (for this, see equation (41)), and that $\frac{\lambda_c^{p\prime}}{\mathrm{u}'\left(c_e^{p\prime}\right)}$ is measurable with respect to the current period's information set (compare Section A.2.3), we have that

$$\kappa_v \frac{\theta^p}{f^p} = \mathbb{E}_a \beta \frac{\lambda_c^{p\prime}}{\lambda_c^p} J^{p\prime}$$

$$-\frac{\psi_s}{f^p(1-s^p)} \frac{1}{\lambda_c^p} \frac{(1-e^p)}{[\xi^p e^p + (1-e^p)]} \frac{\left[1 - \frac{\mathtt{u}'(c_e^p)}{\mathtt{u}'(c_u^p)}\right]}{\frac{\mathtt{u}'(c_e^p)}{\mathtt{u}'(c_u^p)}(1-e^p) + e^p}$$

$$+\frac{\psi_s}{f^p(1-s^p)} \frac{1}{s^p f^p[\xi^p e^p + (1-e^p)]} \frac{1}{\lambda_c^p} e^{p\prime} \tag{62}$$

$$* \left[\frac{1}{\frac{\mathtt{u}'(c_e^p)}{\mathtt{u}'(c_u^p)}(1-e^p) + e^p} - \frac{1}{\frac{\mathtt{u}'\left(c_e^{p\prime}\right)}{\mathtt{u}'\left(c_u^{p\prime}\right)}(1-e^{p\prime}) + e^{p\prime}} \right]. \tag{63}$$

Define, as a new variable, the wedge ζ^p as follows

$$\zeta^p := \frac{\psi_s}{f^p(1-s^p)} \frac{1}{\lambda_c^p} \frac{(1-e^p)}{[\xi^p e^p + (1-e^p)]} \frac{\left[1 - \frac{\mathtt{u}'(c_e^p)}{\mathtt{u}'(c_u^p)}\right]}{\frac{\mathtt{u}'(c_e^p)}{\mathtt{u}'(c_u^p)}(1-e^p) + e^p} \tag{64}$$

$$+\frac{\psi_s}{f^p(1-s^p)} \frac{1}{s^p f^p[\xi^p e^p + (1-e^p)]} \frac{1}{\lambda_c^p} e^{p\prime} \left[\frac{1}{\frac{\mathtt{u}'\left(c_e^{p\prime}\right)}{\mathtt{u}'\left(c_u^{p\prime}\right)}(1-e^{p\prime}) + e^{p\prime}} - \frac{1}{\frac{\mathtt{u}'(c_e^p)}{\mathtt{u}'(c_u^p)}(1-e^p) + e^p} \right].$$

The wedge ζ provides a measure of the lack of insurance. Note that $\zeta \geq 0$, and that ζ is measurable in the current period. The more unequal the consumption of the unemployment and the employment in the planner's allocation, the larger will be ζ (first row). The second row is related to how the degree of insurance evolves over time. An alternative representation of ζ, based on equation (61), is intriguing as well:

$$\zeta^p = \frac{\psi_s}{f^p(1-s^p)} \frac{1}{[\xi^p e^p + (1-e^p)]} \frac{1}{\lambda_c^p} \left[\frac{\lambda_c^p}{\mathtt{u}'(c_e^p)} - 1 \right]$$

$$+\frac{\psi_s}{f^p(1-s^p)} \frac{1}{s^p f^p[\xi^p e^p + (1-e^p)]} \frac{1}{\lambda_c^p} e^{p\prime} \left[\frac{\lambda_c^{p\prime}}{\mathtt{u}'(c_e^{p\prime})} - \frac{\lambda_c^p}{\mathtt{u}'(c_e^p)} \right], \tag{65}$$

showing that the wedge opens up to the extent that the planner's marginal welfare with respect to income is larger than the marginal utility of consumption of the employed worker. Using this definition of ζ^p in equation (63), we derive the following "*free-entry condition in the planner's problem*:"

$$\kappa_v \frac{\theta^p}{f^p} = \mathbb{E}_a \beta \frac{\lambda_c^{p\prime}}{\lambda_c^p} J^{p\prime} - \zeta^p. \tag{66}$$

A.3 Decentralization

This section decentralizes the equilibrium allocation in the planner's problem by means of a set of benefit and tax rules. We proceed in three steps. First, as an intermediate step, we manipulate the firm's value and the worker's surplus in the decentralized economy into a form that will be useful for the proof of Proposition 1. Second, we define a decentralized equilibrium. This definition collects all equilibrium conditions in the decentralized economy. Third, we state Proposition 1. The proposition states that certain tax rules decentralize the planner's allocation. The proof of the Proposition follows. In order to show the equivalence and save on notation, this section uses t-notation throughout.

A.3.1 Preliminaries: value of firm and worker's surplus in the decentralized economy

We next rewrite the firm's value and the worker's surplus in a form that will be useful for proving Proposition 1.

A firm's value in the decentralized economy

We start with the former. In the decentralized economy, the value of the firms is given by equation (9), repeated here for convenience:

$$\begin{aligned} J_t &= -\int_{\epsilon_t^\xi}^{\infty} [\tau_{\xi,t} + w_{eu,t}]\, dF_\epsilon(\epsilon_j) \\ &\quad + \int_{-\infty}^{\epsilon_t^\xi} [\exp\{a_t\} - \epsilon_j - w_t - \tau_{J,t} + \mathbb{E}_t Q_{t,t+1} J_{t+1}]\, dF_\epsilon(\epsilon_j). \end{aligned}$$

Using the properties of the logistic distribution, and the definition $\Psi_\xi(\xi_t) := -\psi_\epsilon[(1-\xi_t)\log(1-\xi_t) + \xi_t \log(\xi_t)]$, we have

$$\begin{aligned} J_t &= (1-\xi_t)\,[\exp\{a_t\} - \mu_\epsilon - w_t - \tau_{J,t} + \mathbb{E}_t Q_{t,t+1} J_{t+1}] \\ &\quad -\xi_t\,[\tau_{\xi,t} + w_{eu,t}] - (1-\xi_t)\mu_\epsilon + \Psi_\xi(\xi_t). \end{aligned}$$

Using the separation probability, equation (8), and otherwise the same steps that – in our write-up of the planner's problem – led to equation (51), we can rewrite $\Psi_\xi(\xi_t)$ as

$$\Psi_\xi(\xi_t) = -\psi \log(1-\xi_t) + \xi_t(\epsilon_t^\xi - \mu_\epsilon).$$

Now use the cutoff value ϵ_t^ξ from equation (17), repeated here as

$$\epsilon_t^\xi = \exp\{a_t\} - \tau_{J,t} + \tau_{\xi,t} + \mathbb{E}_t Q_{t,t+1} J_{t+1} + \frac{\beta \mathbb{E}_t \Delta_{u,t+1}^e + \psi_s \log(1-s_t) - \overline{h}}{\mathrm{u}'(c_{e,t})},$$

to obtain

$$
\begin{aligned}
J_t \quad = \quad & (1-\xi_t)\left[\exp\{a_t\} - \mu_\epsilon - w_t - \tau_{J,t} + \mathbb{E}_t Q_{t,t+1} J_{t+1}\right] \\
& -\xi_t\left[\tau_{\xi,t} + w_{eu,t}\right] - \psi_\epsilon \log(1-\xi_t) \\
& +\xi_t \left[\exp\{a_t\} - \mu_\epsilon - \tau_{J,t} + \tau_{\xi,t} + \mathbb{E}_t Q_{t,t+1} J_{t+1} \right. && (67) \\
& \left. \qquad + \frac{\beta \mathbb{E}_t \Delta^e_{u,t+1} + \psi_s \log(1-s_t) - \overline{h}}{\mathtt{u}'(c_{e,t})}\right] && (68)
\end{aligned}
$$

Recalling that in equilibrium $w_t = w_{eu,t}$, we can simplify this further to

$$
\begin{aligned}
J_t \quad = \quad & \left[\exp\{a_t\} - \mu_\epsilon - w_t - \tau_{J,t} + \mathbb{E}_t Q_{t,t+1} J_{t+1}\right] \\
& -\psi_\epsilon \log(1-\xi_t) + \xi_t \left[\frac{\beta \mathbb{E}_t \Delta^e_{u,t+1} + \psi_s \log(1-s_t) - \overline{h}}{\mathtt{u}'(c_{e,t})}\right]. \qquad (69)
\end{aligned}
$$

Worker's surplus in decentralized economy

Next, we derive a tractable form for the worker's surplus,$\Delta^e_{u,t} := V_{e,t} - V_{u,t}$. Using the value of employment, equation (3), we have that (using that $w_t = w_{eu,t}$ as above)

$$
\Delta^e_{u,t} := V_{e,t} - V_{u,t} \quad = \quad (1-\xi_t)\left[\mathtt{u}(c_{e,t}) + \beta \mathbb{E}_t V_{e,t+1} - V_{u,t}\right] + \xi_t \left[\mathtt{u}(c_{e,t}) - \mathtt{u}(c_{u,t})\right].
$$

The value of unemployment, equation (6), can be rewritten as

$$
V_{u,t} \quad = \quad \mathtt{u}(c_{u,t}) + \overline{h} - \textstyle\int_{-\infty}^{\iota^s_t} \iota_i \, dF_\iota(\iota_i) + s_t f_t \, \beta \mathbb{E}_t \Delta^e_{u,t+1} + \beta \mathbb{E}_t V_{u,t+1}.
$$

Using this,

$$
\begin{aligned}
\Delta^e_{u,t} : \quad = \quad & (1-\xi_t)\left[\mathtt{u}(c_{e,t}) - \mathtt{u}(c_{u,t}) - \overline{h} + \beta \mathbb{E}_t \Delta^e_{u,t+1}\right] + \xi_t\left[\mathtt{u}(c_{e,t}) - \mathtt{u}(c_{u,t})\right] \\
& -(1-\xi_t)\left[-\textstyle\int_{-\infty}^{\iota^s_t} \iota_i \, dF_\iota(\iota_i) + s_t f_t \, \beta \mathbb{E}_t \Delta^e_{u,t+1}\right] \\
= \quad & \mathtt{u}(c_{e,t}) - \mathtt{u}(c_{u,t}) - (1-\xi_t)\overline{h} + (1-\xi_t)(1-s_t f_t)\beta \mathbb{E}_t \Delta^e_{u,t+1} \\
& +(1-\xi_t)\left[\textstyle\int_{-\infty}^{\iota^s_t} \iota_i \, dF_\iota(\iota_i)\right].
\end{aligned}
$$

Next, observe that $-\int_{-\infty}^{\iota^s_t} \iota_i \, dF_\iota(\iota_i) = \Psi_s(s_t) := -\psi_s[(1-s_t)\log(1-s_t) + s_t \log(s_t)]$. Using the search cutoff, equation (4), and the search intensity, equation (5), we have that $\log(\,(1-s_t)/s_t\,) = -f_t \, \beta \, \mathbb{E}_t \Delta^e_{u,t+1}$. We therefore get that

$$
\begin{aligned}
-s_t f_t \, \beta \, \mathbb{E}_t \Delta^e_{u,t+1} - \Psi_s(s_t) \quad = \quad & s_t \psi_s \log(1-s_t) \\
& -s_t \psi_s \log(s_t) + \psi_s[(1-s_t)\log(1-s_t) + s_t \log(s_t)] \\
= \quad & \psi_s \log(1-s_t).
\end{aligned}
$$

Using this, we have

$$\Delta^e_{u,t} = \mathtt{u}(c_{e,t}) - \overline{h}(1-\xi_{,t}) - \mathtt{u}(c_{u,t}) + \beta E_a \Delta^e_{u,t+1}(1-\xi_t) + (1-\xi_t)\psi_s \log(1-s_t). \quad (70)$$

A.3.2 Definition: decentralized equilibrium

A decentralized equilibrium is a sequence of job-finding rates f_t, vacancy-filling rates q_t, separation rates and separation cutoff levels ξ_t and ϵ^ξ_t, search intensities s_t, labor market tightness θ_t, matches m_t, vacancies v_t, consumption levels $c_{e,t}$, $c_{0,t}$, and and $c_{u,t}$, aggregate levels of output y_t, and dividends Π_t, a discount factor $Q_{t,t+1}$, wage rates w_t, employment rates e_t, firm values J_t, and surpluses of the worker Δ_t, and a sequence of government policies (a profit tax rate, $\tau_{J,t}$, a vacancy subsidy, $\tau_{v,t}$, a layoff tax $\tau_{\xi,t}$ and unemployment benefits B_t) such that the following are true:

1. The value of the firm is given by (69), repeated below:

$$\begin{aligned} J_t \quad = \quad & [\exp\{a_t\} - \mu_\epsilon - w_t - \tau_{J,t} + \mathbb{E}_t Q_{t,t+1} J_{t+1}] \\ & -\psi_\epsilon \log(1-\xi_t) + \xi_t \left[\frac{\beta \mathbb{E}_t \Delta^e_{u,t+1} + \psi_s \log(1-s_t) - \overline{h}}{\mathtt{u}'(c_{e,t})} \right]. \end{aligned} \quad (71)$$

2. The surplus of the worker is given by (70), repeated below:

$$\Delta^e_{u,t} = \mathtt{u}(c_{e,t}) - \overline{h}(1-\xi_{,t}) - \mathtt{u}(c_{u,t}) + \beta E_a \Delta^e_{u,t+1}(1-\xi_t) + (1-\xi_t)\psi_s \log(1-s_t). \quad (72)$$

3. The search intensity is optimally chosen, and combining equations (4) and (5), given by:

$$s_t = \frac{1}{1 + e^{\frac{-f_t \beta \mathbb{E}_t \Delta^e_{u,t+1}}{\psi_s}}}. \quad (73)$$

4. Firms choose the number of vacancies optimally, the free-entry condition equation (10) is repeated below:

$$\kappa_v \frac{\theta_t}{f_t}(1-\tau_{v,t}) = \mathbb{E}_t Q_{t,t+1} J_{t+1}. \quad (74)$$

5. Wages w_t, severance payments, $w_{eu,t}$, and separation cutoffs, ϵ^ξ_t, are bargained according to Nash-bargaining protocol (15). The resulting first-order conditions are:

   (a) For the wage, equation (16), repeated here:

$$(1-\eta_t) J_t = \eta_t \frac{\Delta^e_{u,t}}{\mathtt{u}'(c_{e,t})}. \quad (75)$$

(b) For the separation cutoff, equation (17), repeated here:

$$\epsilon_t^{\xi} = [\exp\{a_t\} - \tau_{J,t} + \tau_{\xi,t} + \mathbb{E}_t Q_{t,t+1} J_{t+1} + \frac{\beta \mathbb{E}_t \Delta^e_{u,t+1} + \psi_s \log(1-s_t) - \bar{h}}{\mathtt{u}'(c_{e,t})}. \tag{76}$$

6. The separation cutoff implies a share of separations, ξ_t, that is in line with the logistic distribution. The corresponding equation (8) is repeated here:

$$\xi_t = 1/(1 + \exp\{(\epsilon_t^{\xi} - \mu_\epsilon)/\psi_\epsilon\}. \tag{77}$$

7. Matches link vacancies and workers who search for a job according to matching function (11), repeated here:

$$m_t = \chi v_t^{\gamma} \left[[\xi_t e_t + u_t] s_t\right]^{1-\gamma} \tag{78}$$

8. The job-finding rate is defined as

$$f_t = m_t/(s_t[\xi_t e_t + u_t]). \tag{79}$$

9. The vacancy filling rate is defined as

$$q_t = m_t/v_t. \tag{80}$$

10. Labor-market tightness is defined as

$$\theta_t = v_t/u_t. \tag{81}$$

11. Employment evolves according to (12), or alternatively:

$$e_{t+1} = (1 - \xi_t)e_t + s_t f_t[\xi_t e_t + u_t]. \tag{82}$$

12. Firms price future cash flow using discount factor $Q_{t,t+1} := \beta \frac{\lambda_{t+1}}{\lambda_t}$, where λ_t is given by equation (7):

$$\lambda_t = \frac{\mathtt{u}'(c_{e,t})\mathtt{u}'(c_{u,t})}{\mathtt{u}'(c_{e,t})(1-e_t) + e_t \mathtt{u}'(c_{u,t})}. \tag{83}$$

13. Dividends are given by equation (14), which can be rewritten as

$$\Pi_t = e_t(1-\xi_t)[\exp\{a_t\} - \mu_\epsilon - \tau_{J,t}] - e_t w_t - e_t \xi_t \tau_{\xi,t} + e_t \Psi(\xi_t) - \kappa_v(1-\tau_{v,t})v_t. \tag{84}$$

14. Consumption is given by (2), namely

(a) Consumption when employed during the period:

$$c_{e,t} = w_t + \Pi_t. \tag{85}$$

(b) Consumption when laid-off at the beginning of the period:

$$c_{0,t} = w_t + \Pi_t. \tag{86}$$

(c) Consumption when unemployed at the beginning of the period:

$$c_{u,t} = b_t + \Pi_t. \tag{87}$$

15. Production equals demand (goods markets clear)

$$y_t = e_t(1-\xi_t)\exp\{a_t\}. \tag{88}$$

$$y_t + \mathbf{B}_F(u_t) - \tau_F = e_t c_{e,t} + u_t c_{u,t} + \mu_\epsilon(1-\xi_t)e_t - e_t\Psi_\epsilon(\xi_t) + \kappa_v v_t \tag{89}$$

16. The government budget constraint, equation (27), holds:

$$e_t(1-\xi_t)\tau_{J,t} + e_t\xi_t\tau_{\xi,t} + \mathbf{B}_F(u_t) - \tau_F = u_t b_t + \kappa_v \tau_{v,t} v_t. \tag{90}$$

A.4 Government policies that decentralize planner's allocation

First, let us restate in t-notation the two wedges defined earlier in equations (59) and (64):

$$\varsigma_t = \frac{e_t(1-\xi_t)}{[\xi_t e_t + (1-e_t)]f_t s_t}\left[1 - \frac{\frac{\mathrm{u}'(c_{e,t+1})}{\mathrm{u}'(c_{u,t+1})}(1-e_{t+1}) + e_{t+1}}{\frac{\mathrm{u}'(c_{e,t})}{\mathrm{u}'(c_{u,t})}(1-e_t) + e_t}\right]. \tag{91}$$

$$\begin{aligned}\zeta_t \quad := \quad & \frac{\psi_s}{f_t(1-s_t)}\frac{1}{\lambda_t}\frac{1-e_t}{[\xi_t e_t + (1-e_t)]}\frac{1-\frac{\mathrm{u}'(c_{e,t})}{\mathrm{u}'(c_{u,t})}}{\frac{\mathrm{u}'(c_{e,t})}{\mathrm{u}'(c_{u,t})}(1-e_t)+e_t} \\ & + \frac{\psi_s}{f_t(1-s_t)}\frac{1}{s_t f_t[\xi_t e_t + (1-e_t)]}\frac{1}{\lambda_t}e_{t+1}\left[\frac{1}{\frac{\mathrm{u}'(c_{e,t+1})}{\mathrm{u}'(c_{u,t+1})}(1-e_{t+1})+e_{t+1}} - \frac{1}{\frac{\mathrm{u}'(c_{e,t})}{\mathrm{u}'(c_{u,t})}(1-e_t)+e_t}\right].\end{aligned} \tag{92}$$

The following proposition summarizes the tax and benefit rules that support the social's planner allocation.

Proposition 1. *Consider the economy described in Section 2.1. Consider preferences for which the ratio of marginal utilities in the two states of employment can be expressed as some function g, the sole argument of which is the consumption-based replacement rate $\widetilde{b_t} := c_{u,t}/c_{e,t}$. Namely, $\frac{\mathrm{u}'(c_{e,t})}{\mathrm{u}'(c_{u,t})} = g(\widetilde{b_t})$. Define $\Omega_t := \frac{\eta_t}{\gamma}\frac{1-\gamma}{1-\eta_t}$. In addition, assume that the* *bargaining power η_t is measurable $t-1$. Assume that the values of the tuple of initial states (b_0, a_0, e_0) is the same in the decentralized economy and in the planner's problem described in Appendix A.1. Suppose, in addition, that the government implements the following policies for all periods $t \geq 0$:*

$$\tau_{v,t} = \left[1 - \frac{\Omega_{t+1}}{1+\varsigma_t}\right] + \frac{\eta_{t+1}}{1-\eta_{t+1}} \frac{\zeta_t}{(1+\varsigma_t)\kappa_v \frac{\theta_t}{f_t}}, \tag{93}$$

$$\tau_{\xi,t} = \tau_{J,t} + \tau_{v,t}\kappa_v \frac{\theta_t}{f_t} + \zeta_t(1 - s_t f_t), \tag{94}$$

$$\begin{aligned} \widetilde{b_{t+1}}\mathbb{E}_t\left[\beta\frac{\lambda_{t+1}}{\lambda_t}\frac{c_{e,t+1}}{e_{t+1}}\right] = {} & \tau_{v,t}\kappa_v\frac{\theta_t}{f_t} + \zeta_t - \mathbb{E}_t\left[\beta\frac{\lambda_{t+1}}{\lambda_t}\zeta_{t+1}(1-s_{t+1}f_{t+1})(1-\xi_{t+1})\right] \\ & -\mathbb{E}_t\left[\beta\frac{\lambda_{t+1}}{\lambda_t}\tau_{v,t+1}\kappa_v\frac{\theta_{t+1}}{f_{t+1}}\frac{e_{t+2}}{e_{t+1}}\right] \\ & +\mathbb{E}_t\left[\beta\frac{\lambda_{t+1}}{\lambda_t}\frac{\Pi_{t+1} + \mathbf{B}_F(u_{t+1}) - \tau_F + e_{t+1}\mathbf{B}'_F(u_{t+1})}{e_{t+1}}\right], \end{aligned} \tag{95}$$

$$\begin{aligned} \tau_{J,t} = {} & \kappa_v\tau_{v,t}\theta_t\left[\frac{\xi_t(s_t f_t - 1) + f_t s_t \frac{1-e_t}{e_t}}{f_t}\right] - \xi_t\zeta_t(1-s_t f_t) \\ & + \frac{u_t b_t - \mathbf{B}_F(u_t) + \tau_F}{e_t} \end{aligned} \tag{96}$$

where the two wedges, ς_t and ζ_t, are given by equations (91) *and* (92)*. The following is true*

1. *These tax rules are consistent with the government's budget constraint. That is, the tax rules implement a decentralized equilibrium.*

2. *The equilibrium allocations in the decentralized equilibrium satisfy the first-order conditions in the planner's problem and* vice versa*.*

A.4.1 Proof of Proposition 1

The planner's allocation is characterized by five first-order conditions:

1. With respect to separations: equation (44),
2. With respect to market tightness (hiring): equation (66),
3. With respect to future promised utility: equation (60),
4. With respect to consumption when employed: equation (41),
5. With respect to consumption when unemployed: equation (40),

and four constraints

1. The budget constraint: equation (36),
2. The participation constraint: equation (37),
3. The promise-keeping constraint: equation (38),
4. The law of motion for employment: equation (39).

The proof proceeds by guessing that under the rules stated in Proposition 1 the decentralized economy is characterized by the same allocations as the planner's solution, and verifying that claim.
In particular, we show that under the allocation in the decentralized economy the planner's first-order conditions and constraints are satisfied. Naturally, doing the proof in reverse order the opposite would also be true. In addition, we show that the tax rules balance the budget.
As in the previous text, let the allocation in the planner's economy be marked by a superscript ^{p} whereas the allocation in the decentralized economy carries no such superscript.

Step 1 (Constraints):

Suppose that the allocations in the decentralized equilibrium are the same as in the planner's problem. The four constraints named above figure identically in the decentralized equilibrium. The constraints in the planner's problem are therefore satisfied whenever the allocation is the same as in the decentralized equilibrium.

Step 2 (Vacancy subsidies):

The steps that follow from here onward show that the planner's first-order conditions for separations, market tightness and future promised utility are satisfied as well. In doing so, they substitute in previous equations for the lagrange multipliers that are pinned down by the two first-order conditions for consumption when (un)employed, equations (41) and (40). We start by using the "bargaining equations" in the two economies.
Use the planner's "bargaining equation" (60) and the planner's "free-entry condition" (66), repeated here

$$\kappa_v \frac{\theta_t^p}{\gamma f_t^p} = \mathbb{E}_t \beta \frac{\lambda_{c,t+1}^p}{\lambda_{c,t}^p} J_{t+1}^p + (1+\varsigma_t^p)\mathbb{E}_t \beta \frac{\lambda_{c,t+1}^p}{\lambda_{c,t}^p} \frac{\Delta_{t+1}^p}{\mathrm{u}'(c_{e,t+1}^p)},$$

and

$$\kappa_v \frac{\theta_t^p}{f_t^p} = \mathbb{E}_t \beta \frac{\lambda_{c,t+1}^p}{\lambda_{c,t}^p} J_{t+1}^p - \zeta_t^p.$$

Merge these two equations by substituting for $\mathbb{E}_t \beta \frac{\lambda_{c,t+1}^p}{\lambda_{c,t}^p} J_{t+1}^p$:

$$\kappa_v \frac{\theta_t^p}{\gamma f_t^p} - (1+\varsigma_t^p)\mathbb{E}_t \beta \frac{\lambda_{c,t+1}^p}{\lambda_{c,t}^p} \frac{\Delta_{t+1}^p}{\mathrm{u}'(c_{e,t+1}^p)} = \kappa_v \frac{\theta_t^p}{f_t^p} + \zeta_t^p.$$

Rearrange:

$$\frac{\kappa_v \frac{\theta_t^p}{f_t^p} \frac{1-\gamma}{\gamma}}{1+\varsigma_t^p} = \mathbb{E}_t \beta \frac{\lambda_{c,t+1}^p}{\lambda_{c,t}^p} \frac{\Delta_{t+1}^p}{\mathrm{u}'(c_{e,t+1}^p)} + \frac{\zeta_t^p}{1+\varsigma_t^p}. \tag{97}$$

Now compare equation (97) to the free entry-condition (98) in the decentralized equilibrium. Into the latter, substitute the bargaining condition (75) and the definition of the discount factor in the decentralized economy to obtain:

$$\kappa_v \frac{\theta_t}{f_t}(1-\tau_{v,t}) = \mathbb{E}_t \beta \frac{\lambda_{t+1}}{\lambda_t} \frac{\eta_{t+1}}{(1-\eta_{t+1})} \frac{\Delta_{u,t+1}^e}{\mathrm{u}'(c_{e,t+1})}.$$

Use that, by assumption, η_{t+1} is measurable t

$$\kappa_v \frac{\theta_t}{f_t}(1-\tau_{v,t}) = \frac{\eta_{t+1}}{(1-\eta_{t+1})} \mathbb{E}_t \beta \frac{\lambda_{t+1}}{\lambda_t} \frac{\Delta_{u,t+1}^e}{\mathrm{u}'(c_{e,t+1})}. \tag{98}$$

Now, observe that if the allocations are the same then $\lambda_t = \lambda_{c,t}^p$; compare equations (83) and (41). Also observe that if the allocations are the same then $\zeta_t^p = \zeta_t$ and $\varsigma_t^p = \varsigma_t$; compare equations (91) and (92). Last, observe that if the allocations are the same then $\Delta_{u,t}^e = \Delta_t^p$.

As a result, with the same allocation equations (97) and (98) can only be true at the same time if

$$\frac{\kappa_v \frac{\theta_t}{f_t} \frac{1-\gamma}{\gamma}}{1+\varsigma_t} = \kappa_v \frac{\theta_t}{f_t}(1-\tau_{v,t}) \frac{(1-\eta_{t+1})}{\eta_{t+1}} + \frac{\zeta_t}{1+\varsigma_t}.$$

It is straightforward to show that the vacancy subsidy (93) ensures that this the above equation holds.

Step 3 (Benefits):

This step compares the planner's and the decentralized economy's "free-entry conditions." The planner's free entry condition (66) is:

$$\kappa_v \frac{\theta_t^p}{f_t^p} = \beta \mathbb{E}_t \left[\frac{\lambda_{c,t+1}^p}{\lambda_{c,t}^p} J_{t+1}^p \right] - \zeta_t^p. \tag{99}$$

The decentralized economy's free-entry condition (98) is

$$\kappa_v \frac{\theta_t}{f_t} = \beta \mathbb{E}_t \left[\frac{\lambda_{t+1}}{\lambda_t} J_{t+1} \right] + \kappa_v \frac{\theta_t}{f_t} \tau_{v,t}.$$

So, if the allocations are the same in the two economies,

$$\beta \mathbb{E}_t \left[\frac{\lambda_{c,t+1}^p}{\lambda_{c,t}} J_{t+1}^p \right] - \beta \mathbb{E}_t \left[\frac{\lambda_{c,t+1}}{\lambda_{c,t}} J_{t+1} \right] = \kappa_v \frac{\theta_t}{f_t} \tau_{v,t} + \zeta_t^p. \tag{100}$$

Next, we develop an expression for the two terms on the left-hand side.

The planner's "profit equation" is given by (53) and the profit equation in the decentralized economy is (71). These are repeated here:

$$\begin{aligned} J_t^p \;=\; & \exp\{a_t\} - \mathbf{B}_F'(u_t^p) - \mu_\epsilon - c_{e,t}^p + c_{u,t}^p - \psi_\epsilon \log(1-\xi_t^p) \\ & +\kappa_v s_t^p \theta_t^p + \frac{\xi_t^p}{\mathtt{u}'(c_{e,t}^p)}\left[\mathbb{E}_t \beta \Delta_{t+1}^p - \overline{h}\right] + \frac{\xi_t^p}{\mathtt{u}'(c_{e,t}^p)}\psi_s \log(1-s_t^p) \\ & +\mathbb{E}_t \beta \frac{\lambda_{c,t+1}^p}{\lambda_{c,t}^p} J_{t+1}^p (1 - s_t^p f_t^p), \end{aligned}$$

and

$$\begin{aligned} J_t \;=\; & \left[\exp\{a_t\} - \mu_\epsilon - w_t - \tau_{J,t} + \beta \mathbb{E}_t \frac{\lambda_{t+1}}{\lambda_t} J_{t+1}\right] \\ & -\psi_\epsilon \log(1-\xi_t) + \xi_t \left[\frac{\beta \mathbb{E}_t \Delta_{u,t+1}^e + \psi_s \log(1-s_t) - \overline{h}}{\mathtt{u}'(c_{e,t})}\right]. \end{aligned}$$

Notice that $c_{e,t} = w_t + \Pi_t$ with Π_t being dividends, and $c_{u,t} = b_t + \Pi_t$. Set equal the allocations in the two economies. Use that for the same allocations $\Delta_{u,t}^e = \Delta_t^p$ and $\lambda_t = \lambda_{c,t}^p$. We have that

$$J_t - J_t^p = \quad -b_t + \mathbf{B}_F'(u_t^p) - \tau_{J,t} + \beta \mathbb{E}_t \frac{\lambda_{t+1}}{\lambda_t}\left[J_{t+1} - J_{t+1}^p\right] - \kappa_v s_t \theta_t + \mathbb{E}_t \beta \frac{\lambda_{t+1}}{\lambda_t} J_{t+1}^p s_t f_t.$$

Using the planner's free-entry condition:

$$J_t - J_t^p = \mathbf{B}_F'(u_t^p) - b_t - \tau_{J,t} + \beta \mathbb{E}_t \frac{\lambda_{t+1}}{\lambda_t}\left[J_{t+1} - J_{t+1}^p\right] + s_t f_t \zeta_t.$$

Using the difference between the two free-entry conditions from above,

$$J_t - J_t^p = \mathbf{B}_F'(u_t^p) - b_t - \tau_{J,t} - \kappa_v \frac{\theta_t}{f_t} \tau_{v,t} - \zeta_t (1 - s_t f_t).$$

Moving this forward by one period, augmenting appropriately and taking expectations,

$$\beta \mathbb{E}_t \frac{\lambda_{t+1}}{\lambda_t}\left[J_{t+1} - J_{t+1}^p\right] = \beta \mathbb{E}_t \frac{\lambda_{t+1}}{\lambda_t}\left[\mathbf{B}_F'(u_{t+1}^p) - b_{t+1} - \tau_{J,t+1} - \kappa_v \frac{\theta_{t+1}}{f_{t+1}} \tau_{v,t+1} - \zeta_{t+1}(1 - s_{t+1} f_{t+1})\right].$$

Combining this with the difference in the free-entry conditions (100) we have

$$\beta \mathbb{E}_t \frac{\lambda_{t+1}}{\lambda_t}\left[\mathbf{B}_F'(u_{t+1}) - b_{t+1} - \tau_{J,t+1} - \kappa_v \frac{\theta_{t+1}}{f_{t+1}} \tau_{v,t+1} - \zeta_{t+1}(1 - s_{t+1} f_{t+1})\right] = -\kappa_v \frac{\theta_t}{f_t} \tau_{v,t} - \zeta_t.$$

Rearrange to get

$$\begin{aligned}\beta\mathbb{E}_t\tfrac{\lambda_{t+1}}{\lambda_t}b_{t+1} &= -\beta\mathbb{E}_t\tfrac{\lambda_{t+1}}{\lambda_t}\left[\tau_{J,t+1}-\mathbf{B}_F'(u_{t+1})+\kappa_v\tfrac{\theta_{t+1}}{f_{t+1}}\tau_{v,t+1}+\zeta_{t+1}(1-s_{t+1}f_{t+1})\right]\\ &\quad +\kappa_v\tfrac{\theta_t}{f_t}\tau_{v,t}+\zeta_t.\end{aligned}$$

The next lines rewrite this equation as an expression for the replacement rate in $t+1$, which we know from Appendix A.2.3 is measurable t. To rearrange, use the government budget constraint, equation (90), divided by employment:

$$(1-\xi_t)\tau_{J,t}+\xi_t\tau_{\xi,t}=\frac{u_tb_t-\mathbf{B}_F(u_t)+\tau_F}{e_t}+\kappa_v\tau_{v,t}s_t\theta_t\left[\xi_t+\frac{1-e_t}{e_t}\right].$$

Also use the postulated evolution for the layoff tax, equation (94)

$$\tau_{J,t}=-\xi_t\tau_{v,t}\kappa_v\frac{\theta_t}{f_t}-\xi_t\zeta_t(1-s_tf_t)+\frac{u_tb_t-\mathbf{B}_F(u_t)+\tau_F}{e_t}+\kappa_v\tau_{v,t}s_t\theta_t\left[\xi_t+\frac{1-e_t}{e_t}\right]. \quad (101)$$

Use this to substitute out $\tau_{J,t+1}$ above, and also use $c_{u,t}=b_t+\Pi_t$ to get

$$\begin{aligned}\beta\mathbb{E}_t\tfrac{\lambda_{t+1}}{\lambda_t}(c_{u,t+1}-\Pi_{t+1}-\mathbf{B}_F'(u_{t+1})) &= \kappa_v\tfrac{\theta_t}{f_t}\tau_{v,t}+\zeta_t-\beta\mathbb{E}_t\tfrac{\lambda_{t+1}}{\lambda_t}\kappa_v\tfrac{\theta_{t+1}}{f_{t+1}}\tau_{v,t+1}\\ &\quad -\beta\mathbb{E}_t\tfrac{\lambda_{t+1}}{\lambda_t}\zeta_{t+1}(1-s_{t+1}f_{t+1})(1-\xi_{t+1})\\ &\quad -\beta\mathbb{E}_t\tfrac{\lambda_{t+1}}{\lambda_t}\Big[-\xi_{t+1}\tau_{v,t+1}\kappa_v\tfrac{\theta_{t+1}}{f_{t+1}}\\ &\qquad +\kappa_v\tau_{v,t+1}s_{t+1}\theta_{t+1}\left(\xi_{t+1}+\tfrac{1-e_{t+1}}{e_{t+1}}\right)\Big]\\ &\quad -\beta\mathbb{E}_t\tfrac{\lambda_{t+1}}{\lambda_t}\tfrac{u_{t+1}}{e_{t+1}}\left(c_{u,t+1}-\Pi_{t+1}+\tfrac{-\mathbf{B}_F(u_{t+1})+\tau_F}{u_{t+1}}\right).\end{aligned}$$

Simplify further

$$\begin{aligned}\beta\mathbb{E}_t\tfrac{\lambda_{t+1}}{\lambda_t}c_{u,t+1}\tfrac{1}{e_{t+1}} &= \kappa_v\tfrac{\theta_t}{f_t}\tau_{v,t}+\zeta_t\\ &\quad -\beta\mathbb{E}_t\tfrac{\lambda_{t+1}}{\lambda_t}\zeta_{t+1}(1-s_{t+1}f_{t+1})(1-\xi_{t+1})-\beta\mathbb{E}_t\tfrac{\lambda_{t+1}}{\lambda_t}\kappa_v\tfrac{\theta_{t+1}}{f_{t+1}}\tau_{v,t+1}\\ &\quad -\beta\mathbb{E}_t\tfrac{\lambda_{t+1}}{\lambda_t}\left[-\xi_{t+1}\tau_{v,t+1}\kappa_v\tfrac{\theta_{t+1}}{f_{t+1}}+\kappa_v\tau_{v,t+1}s_{t+1}\theta_{t+1}\left(\xi_{t+1}+\tfrac{1-e_{t+1}}{e_{t+1}}\right)\right]\\ &\quad +\beta\mathbb{E}_t\tfrac{\lambda_{t+1}}{\lambda_t}[\tfrac{\Pi_{t+1}+\mathbf{B}_F(u_{t+1})-\tau_F+e_{t+1}\mathbf{B}_F'(u_{t+1})}{e_{t+1}}].\end{aligned}$$

Define $\widetilde{b_{t+1}}:=c_{u,t+1}/c_{e,t+1}$, which for CRRA utility is measurable t by Appendix A.2.3. Then simplify to

$$\begin{aligned}\widetilde{b_{t+1}}\beta\mathbb{E}_t\tfrac{\lambda_{t+1}}{\lambda_t}\tfrac{c_{e,t+1}}{e_{t+1}} &= \kappa_v\tfrac{\theta_t}{f_t}\tau_{v,t}+\zeta_t\\ &\quad -\beta\mathbb{E}_t\tfrac{\lambda_{t+1}}{\lambda_t}\zeta_{t+1}(1-s_{t+1}f_{t+1})(1-\xi_{t+1})-\beta\mathbb{E}_t\tfrac{\lambda_{t+1}}{\lambda_t}\kappa_v\tfrac{\theta_{t+1}}{f_{t+1}}\tau_{v,t+1}(1-\xi_{t+1})\\ &\quad -\beta\mathbb{E}_t\tfrac{\lambda_{t+1}}{\lambda_t}\left[\kappa_v\tau_{v,t+1}\tfrac{\theta_{t+1}}{f_{t+1}}\left(\xi_{t+1}+\tfrac{1-e_{t+1}}{e_{t+1}}\right)s_{t+1}f_{t+1}\right]\\ &\quad +\beta\mathbb{E}_t\tfrac{\lambda_{t+1}}{\lambda_t}\tfrac{\Pi_{t+1}+\mathbf{B}_F(u_{t+1})-\tau_F+e_{t+1}\mathbf{B}_F'(u_{t+1})}{e_{t+1}}.\end{aligned}$$

Use the employment flow equation, namely $e_{t+1} - e_t(1-\xi_t) = [\xi_t e_t + (1-e_t)]s_t f_t$, to get

$$\begin{aligned}\widetilde{b_{t+1}}\beta\mathbb{E}_t\tfrac{\lambda_{t+1}}{\lambda_t}\tfrac{c_{e,t+1}}{e_{t+1}} \quad = \quad & \kappa_v\tfrac{\theta_t}{f_t}\tau_{v,t} + \zeta_t \\ & -\beta\mathbb{E}_t\tfrac{\lambda_{t+1}}{\lambda_t}\zeta_{t+1}(1-s_{t+1}f_{t+1})(1-\xi_{t+1}) - \beta\mathbb{E}_t\tfrac{\lambda_{t+1}}{\lambda_t}\kappa_v\tfrac{\theta_{t+1}}{f_{t+1}}\tau_{v,t+1}(1-\xi_{t+1}) \\ & -\beta\mathbb{E}_t\tfrac{\lambda_{t+1}}{\lambda_t}\left[\kappa_v\tau_{v,t+1}\tfrac{\theta_{t+1}}{f_{t+1}}\tfrac{e_{t+2}-e_{t+1}(1-\xi_{t+1})}{e_{t+1}}\right] \\ & +\beta\mathbb{E}_t\tfrac{\lambda_{t+1}}{\lambda_t}\tfrac{\Pi_{t+1}+\mathbf{B}_F(u_{t+1})-\tau_F+e_{t+1}\mathbf{B}'_F(u_{t+1})}{e_{t+1}},\end{aligned}$$

or

$$\begin{aligned}\widetilde{b_{t+1}}\beta\mathbb{E}_t\tfrac{\lambda_{t+1}}{\lambda_t}\tfrac{c_{e,t+1}}{e_{t+1}} \quad = \quad & \kappa_v\tfrac{\theta_t}{f_t}\tau_{v,t} + \zeta_t \\ & -\beta\mathbb{E}_t\tfrac{\lambda_{t+1}}{\lambda_t}\left[\zeta_{t+1}(1-s_{t+1}f_{t+1})(1-\xi_{t+1}) + \kappa_v\tau_{v,t+1}\tfrac{\theta_{t+1}}{f_{t+1}}\tfrac{e_{t+2}}{e_{t+1}}\right] \\ & +\beta\mathbb{E}_t\tfrac{\lambda_{t+1}}{\lambda_t}\tfrac{\Pi_{t+1}+\mathbf{B}_F(u_{t+1})-\tau_F+e_{t+1}\mathbf{B}'_F(u_{t+1})}{e_{t+1}}\end{aligned}$$

which is equation (95) in the Proposition.

Step 4 (Layoff Tax):

We next show that, using layoff tax (94), the first-order conditions for layoffs are the same in the planner's problem as in the decentralized economy. The private cut-off is given by equation (17), repeated here:

$$\epsilon_t^{\xi} = \exp\{a_t\} - \tau_{J,t} + \tau_{\xi,t} + \mathbb{E}_t Q_{t,t+1}J_{t+1} + \frac{\beta\mathbb{E}_t\Delta_{u,t+1}^e + \psi_s\log(1-s_t) - \overline{h}}{\mathrm{u}'(c_e)}.$$

The planner's cut-off is given by equation (52), rewritten here:

$$\epsilon_t^{\xi,p} = \exp\{a_t\} + \kappa_v s_t^p\theta_t^p + \mathbb{E}_t\beta\frac{\lambda_{c,t+1}^p}{\lambda_{c,t}^p}J_{t+1}^p(1-s_t^p f_t^p) + \frac{\mathbb{E}_t\beta\Delta_{t+1}^p + \psi_s\log(1-s_t^p) - \overline{h}}{\mathrm{u}'(c_{e,t}^p)}.$$

Using the planner's free-entry condition (99),

$$\epsilon_t^{\xi,p} = \exp\{a_t\} - s_t^p f_t^p\zeta_t^p + \mathbb{E}_t\beta\frac{\lambda_{c,t+1}^p}{\lambda_{c,t}^p}J_{t+1}^p + \frac{\mathbb{E}_t\beta\Delta_{t+1}^p + \psi_s\log(1-s_t^p) - \overline{h}}{\mathrm{u}'(c_{e,t}^p)}.$$

If the allocations are to be the same, then $\epsilon_t^{\xi,p}$ needs to equal ϵ_t^{ξ}. Imposing this, and the equality of the allocations, and observing that $\Delta_t^p = \Delta_{u,t}^e$ under these assumptions, we have:

$$-\tau_{J,t} + \tau_{\xi,t} + \beta\mathbb{E}_t\frac{\lambda_{t+1}}{\lambda_t}J_{t+1} = -s_t f_t\zeta_t + \mathbb{E}_t\beta\frac{\lambda_{t+1}}{\lambda_t}J_{t+1}^p.$$

Using equation (100) to substitute for $\beta \mathbb{E}_t \frac{\lambda^p_{t+1}}{\lambda_t} \left[J^p_{t+1} - J_{t+1} \right] = \kappa_v \frac{\theta_t}{f_t} \tau_{v,t} + \zeta_t$

$$\tau_{\xi,t} = \tau_{J,t} + \kappa_v \frac{\theta_t}{f_t} \tau_{v,t} + (1 - s_t f_t)\zeta_t.$$

This is the rule for layoff taxes provided in the proposition, compare equation (94).

Step 5 (Feasibility):

It remains to be shown that the set of tax and benefit rules provided in the proposition is feasible, that is, it fulfills the government's budget constraint in the decentralized economy, equation (90). The latter is given by

$$e_t(1 - \xi_t)\tau_{J,t} + e_t \xi_t \tau_{\xi,t} + \mathbf{B}_F(u_t) - \tau_F = u_t b_t + \kappa_v \tau_v v_t.$$

Using layoff taxes, equation (94)

$$\tau_{\xi,t} = \tau_{J,t} + \tau_{v,t} \kappa_v \frac{\theta_t}{f_t} + \zeta_t (1 - s_t f_t),$$

and rearranging, this can be written as

$$\tau_{J,t} = \kappa_v \tau_{v,t} \theta_t \left[\frac{\xi_t (s_t f_t - 1) + f_t s_t \frac{1-e_t}{e_t}}{f_t} \right] - \xi_t \zeta_t (1 - s_t f_t) + \frac{u_t b_t - \mathbf{B}_F(u_t) + \tau_F}{e_t},$$

which is the rule for the production tax (96) in the proposition.
This concludes the proof of the proposition.

A.5 Proof of the paper's proposition

The following proposition summarizes the tax and benefit rules that support the social's planner allocation in a steady state.

Proposition 2. *Consider the economy described in Section 2.1. Define* $\Omega_t := \frac{\eta_t}{\gamma} \frac{1-\gamma}{1-\eta_t}$ *and* $\zeta = \frac{\psi_s}{f(1-s)} \frac{1-e}{[\xi e + (1-e)]} \frac{\mathrm{u}'(c_u) - \mathrm{u}'(c_e)}{\mathrm{u}'(c_u)\mathrm{u}'(c_e)}$. *Then the following instruments decentralize the planner's* *solution in steady state.*

$$\tau_v = [1-\Omega] + \frac{\eta}{1-\eta}\frac{\zeta}{\kappa_v\frac{\theta}{f}}, \tag{102}$$

$$\tau_\xi = \tau_J + \tau_v\kappa_v\frac{\theta}{f} + \zeta(1-sf), \tag{103}$$

$$\begin{aligned} b &= \frac{(1-\beta)}{\beta}\tau_v\kappa_v\frac{\theta}{f}e + \zeta e\frac{[1-\beta(1-sf)(1-\xi)]}{\beta} \\ &\quad + \mathbf{B}_F(u) - \tau_F + e\mathbf{B}'_F(u), \end{aligned} \tag{104}$$

$$\tau_J = \frac{1-e}{e}[b-\zeta sf] - \frac{\mathbf{B}_F(u)-\tau_F}{e}, \tag{105}$$

<u>Proof:</u> (102) is simply the steady-state version of the corresponding line in the previous proposition. The same is true of (103) and of (104). (105) follows from (96) using the following two steady-state relations

$$e\xi = sf[\xi e + 1 - e]$$

$$\xi(1-sf) = sf\frac{[1-e]}{e}.$$

This concludes the proof of Proposition 2. □

A.6 Definition of micro elasticity $\epsilon_{D_2,b}$

Define the average duration of unemployment as $D \equiv \frac{1}{sf}$. Define the duration over which UI benefits are paid as $D_2 = D - 1$. Further define the micro-elasticity of duration D_2 with respect to c_u as $\epsilon_{D_2,b}$. The thought-experiment behind this elasticity is that for the duration of the unemployment spell benefits rise, but that that the rise in benefits is not expected to be permanent. That is, while the spell lasts, each and every period anew the worker is surprised that benefits are announced to be higher than usual for the next period. In this sense, this is the static micro elasticity of duration with respect to a previously unanticipated increase in benefits.
To derive this elasticity, focus on the case $\beta \to 1$ and of log utility. Observe that the worker's search decision implies

$$fE\Delta' = -\psi_s\left[\log(1-s) - \log(s)\right]$$

So that, taking the derivative with respect to c'_u:

$$fE\left\{\frac{\partial\Delta'}{\partial\log c'_u}\right\} = \frac{\psi_s}{1-s}\frac{\partial s}{\partial\log c'_u} + \frac{\psi_s}{s}\frac{\partial s}{\partial\log c'_u}$$

From the surplus equation, (72), for a one-period change in benefits next period (future changes thereafter not being anticipated),

$$\frac{\partial \Delta'}{\partial \log(c_u')} = -1,$$

so that

$$-f = \frac{\psi_s}{1-s}\frac{\partial s}{\partial \log c_u'} + \frac{\psi_s}{s}\frac{\partial s}{\partial \log c_u'},$$

implying

$$\frac{\partial \log s}{\partial \log c_u'} = -\frac{f(1-s)}{\psi_s}.$$

Again, this is the change in search intensity when benefits are announced to change for one period ahead. The effect of a sequence of such unannounced changes on realized average duration is given by

$$\left.\frac{\partial \log D}{\partial \log c_u'}\right|_{\text{repeated raises}} = \frac{-\partial[\log s + \log f]}{\partial \log c_u'} = -\frac{\partial \log s}{\partial \log c_u'} = \frac{f(1-s)}{\psi_s}.$$

The elasiticity $\epsilon_{D_2,b}$ then is

$$\epsilon_{D_2,b} := \left.\frac{\partial \log D_2}{\partial \log c_u'}\right|_{\text{repeated raises}} = \left.\frac{\partial \log D}{\partial \log c_u'}\right|_{\text{repeated raises}} \frac{D}{D_2} = \frac{f(1-s)}{\psi_s}\frac{D}{D_2},$$

as described in the text.

A.7 Proof of the paper's corollary

Note that, as a special case of Section A.5, we obtain for $\beta \to 1$, log-utility and the Hosios condition holding ($\Omega = 1$):

$$\tau_v = \frac{\eta}{1-\eta}\frac{\zeta}{\kappa_v \frac{\theta}{f}}, \tag{106}$$

$$\tau_\xi = \tau_J + \tau_v \kappa_v \frac{\theta}{f} + \zeta(1 - sf), \tag{107}$$

$$b = \zeta sf + \mathbf{B}_F(u) - \tau_F + e\mathbf{B}_F'(u), \tag{108}$$

$$\tau_J = \frac{1-e}{e}[b - \zeta sf] - \frac{\mathbf{B}_F(u) - \tau_F}{e}, \tag{109}$$

The following uses these relations and provides a proof to the statements in the main text's corollary.

Proposition 3. *Assume that $\beta \to 1$, that there is log-utility and that the Hosios condition holds, then*

1. *The steady state replacement rate is given by a version of the Baily-Chetty-Formula plus the impact of the federal unemployment insurance system expressed relative to average wages w scaled by the elasticity of search* $\epsilon_{D_2,b}$*:*

$$\frac{b}{w} = \underbrace{\frac{1}{1+D\epsilon_{D_2,b}}}_{autarky} + \underbrace{\frac{D\epsilon_{D_2,b}}{1+D\epsilon_{D_2,b}}\left[\frac{\mathbf{B}_F(u)-\tau_F}{w} + e\frac{\mathbf{B}'_F(u)}{w}\right]}_{federal\ UI} \tag{110}$$

2. *Profit taxes feed through one-to-one any federal transfer as a tax rebate, and they increase linearly with the marginal generosity of the federal unemployment reinsurance:*

$$\frac{\tau_J}{w} = \underbrace{-\frac{\mathbf{B}_F(u)-\tau_F}{w} + (1-e)\frac{\mathbf{B}'_F(u)}{w}}_{federal\ UI} \tag{111}$$

3. *Both vacancy subsidies and the separation tax decline linearly with the marginal impact of the European unemployment insurance:*

$$\begin{aligned}\kappa_v\frac{\theta}{f}\frac{\tau_v}{w} &= \frac{\eta}{1-\eta}\underbrace{\frac{D}{1+D\epsilon_{D_2,b}}}_{autarky} \\ &\quad \underbrace{-\frac{\eta}{1-\eta}\frac{D}{1+D\epsilon_{D_2,b}}\left[\frac{\mathbf{B}_F(u)-\tau_F}{w} + e\frac{\mathbf{B}'_F(u)}{w}\right]}_{federal\ UI} \end{aligned}\tag{112}$$

$$\begin{aligned}\frac{\tau_\xi}{w} &= \underbrace{\frac{[\frac{D}{1-\eta}-1]}{1+D\epsilon_{D_2,b}}}_{autarky} \\ &\quad \underbrace{-\frac{\left[D\epsilon_{D_2,b}+\frac{D}{1-\eta}\right]}{1+D\epsilon_{D_2,b}}\cdot\left[\frac{\mathbf{B}_F(u)-\tau_F}{w}\right]}_{federal\ UI} \\ &\quad \underbrace{-\left[\frac{D\epsilon_{D_2,b}+\frac{D}{1-\eta}}{1+D\epsilon_{D_2,b}}e-1\right]\frac{\mathbf{B}'_F(u)}{w}}_{federal\ UI}\end{aligned}\tag{113}$$

4. *Dividends are zero* ($\Pi = 0$)*.*

<u>Proof of item 1</u>. From (108), we have that

$$b = \zeta sf + \mathbf{B}_F(u) - \tau_F + e\mathbf{B}'_F(u)$$

Evaluate equation (92) in steady state (using that $c_e - c_u = w - b$) to get that

$$\zeta = \frac{\psi_s}{f(1-s)} \frac{1-e}{\xi e + (1-e)}(w-b)$$

Use the definition of the elasticity of duration from above to get that

$$\zeta = \frac{1}{\epsilon_{D_2,b}} \frac{D}{D_2} \frac{1-e}{\xi e + (1-e)}(w-b)$$

Thus, using that $D = 1/(sf)$, we have that

$$\zeta sf = \frac{1}{\epsilon_{D_2,b}} \frac{1}{D_2} \frac{1-e}{\xi e + (1-e)}(w-b)$$

Next, observe the following steady state relations

$$\begin{aligned} e\xi &= sf[\xi e + 1 - e] \\ \xi(1-sf) &= sf\frac{[1-e]}{e} \\ \frac{D_2}{D} &= \frac{1-e}{[\xi e + 1 - e]} \end{aligned}$$

Thus, $\frac{1}{D_2}\frac{1-e}{\xi e+(1-e)} = 1/D$, so that $\zeta sf = \frac{1}{\epsilon_{D_2,b}D}(w-b)$. With this, (108) delivers

$$b = \frac{1}{\epsilon_{D_2,b}D}(w-b) + \mathbf{B}_F(u) - \tau_F + e\mathbf{B}'_F(u),$$

or, multiplying by $\epsilon_{D_2,b}D$, dividing by w, and rearranging,

$$b\left[1 + \epsilon_{D_2,b}D\right]/w = 1 + \epsilon_{D_2,b}D\left[\frac{\mathbf{B}_F(u) - \tau_F + e\mathbf{B}'_F(u)}{w}\right],$$

from where (110) follows immediately.

Proof of item 2. This follows directly from (109) after substituting for $b - \zeta sf$ using the results from the proof of item 1.

Proof of item 3. (106) follows from (112) after substituting for the steady state for ζ and using (110). To find (113), solve for ζ from (106). Observing that $1 - sf = 1 - 1/D$, equation (107) can be simplified to give

$$\tau_\xi = \tau_J + \tau_v\kappa_v\frac{\theta}{f}\left[\frac{1}{\eta} - \frac{1-\eta}{\eta}\frac{1}{D}\right].$$

Equation (113) arises when substituting for τ_v and τ_J from (112) and (111) and simplifying. This concludes the proof of item 3.

Proof of item 4. In order to see that dividends are zero in steady state, first observe that in a steady state with $\beta \to 1$ and log utility, (71) implies

$$w = \exp\{a\} - \mu_\epsilon - \tau_J - \psi_s \log(1-\xi) + \xi c_e \left[\Delta_u^e + \psi_s \log(1-s) - \overline{h}\right] \tag{114}$$

Use that, by (76) and (77),

$$\left[\Delta_u^e + \psi_s \log(1-s) - \overline{h}\right] c_e = \psi_\epsilon \log((1-\xi)/\xi) - [\exp(a) - \mu - \tau_J + \tau_\xi + J]$$

Substitute this in (114) and use $J = \kappa_v \theta / f(1-\tau_v)$ (from (98)) to obtain

$$w = (1-\xi)\left[\exp\{a\} - \mu_\epsilon - \tau_J\right] - \xi\tau_\xi - \xi\kappa_v \frac{\theta}{f}(1-\tau_v) - \psi_\epsilon[(1-\xi)\log(1-\xi) + \xi\log(\xi)] \tag{115}$$

Note that by (84) dividends in steady state are given by

$$\Pi = e(1-\xi)[\exp\{a\} - \mu_\epsilon - \tau_J] - ew_t - e\xi\tau_\xi + e\Psi(\xi) - \kappa_v(1-\tau_v)v.$$

Insert (115) for the wage, observe that $\Psi(\xi) = -\psi_\epsilon[(1-\xi)\log(1-\xi) + \xi\log(\xi)]$, and cancel terms to get that

$$\Pi = \kappa_v(1-\tau_v)[e\xi\theta/f - v].$$

Note that the term in square brackets is zero. This is so, since $e\xi = m$ and $\theta/f = v/m$. This proves that profits are zero in the steady state considered here. □

The formulations used in the main text's version of the corollary follow from straightforward rearranging. This is obvious for (32), for example. As another example, (34) follows from equation (107), observing that $1 - sf = 1 - 1/D = D_2/D$ and substituting for ζ from (106). Then use (32).

B Linking the two micro-elasticities of unemployment duration

The current appendix clarifies the relation between the two different ways of defining the micro-elasticity of unemployment duration with respect to benefits that are used in the paper. One of these, we use as a calibration target. The other one occurs in the main text's corollary. In both cases, the micro elasticity of unemployment duration with respect to benefits is given by

$$\frac{\partial \log D}{\partial \log c_u'} = \frac{\partial \log \frac{1}{sf}}{\partial \log c_u'} = -\frac{\partial \log s}{\partial \log c_u'}. \tag{116}$$

The two cases differ by the anticipated duration over which benefits are increased, and thus by the effect of benefits on households' search behavior.

Household search behavior. Combining (4) and (5)

$$f_t \beta E_t \Delta_{t+1} = -\psi_s \log(1 - s_t) + \psi_s \log(s_t).$$

From this

$$\frac{\partial \log s}{\partial \log c_u'} = \frac{f(1-s)}{\psi_s} \beta E \frac{\partial \log \Delta'}{\partial \log c_u'}. \tag{117}$$

Note that, $\Delta_t = V_{e,t} - V_{u,t}$. Solving from (3) and (6) gives

$$\Delta_t = \log(c_{e,t}) - \log(c_{u,t}) - (1 - \xi_t)[\overline{h} - \psi_s \log(1 - s_t) + \beta(1 - \xi_t) E_t \Delta_{t+1}. \tag{118}$$

One-period raises in benefits. The corollary in the main text defines the micro-elasticity of unemployment duration with respect to benefits as the elasticity of repeated, unanticipated one-period increases in benefits. That is, in each period of its unemployment spell, the household learns that unemployment benefits unexpectedly are raised also for the next period. With this, the expected continuation value for Δ on the right-hand side of (118) is not affected, and $\partial \Delta' / \partial \log c_u' = -1$. Then, by (116) and (117)

$$\left. \frac{\partial \log D}{\partial \log c_u'} \right|_{\text{repeated raises}} = \frac{f(1-s)}{\psi_s} \beta.$$

The formula for $\epsilon_{D_2,b}$, derived in Appendix A.6, that also appears in the corollary looks at the elasticity of the duration of unemployment during which the government pays benefits, $D_2 = D - 1$, but otherwise is exactly the above:

$$\epsilon_{D_2,b} := \left. \frac{\partial \log D_2}{\partial \log c_u'} \right|_{\text{repeated raises}} = \frac{D}{D_2} \frac{f(1-s)}{\psi_s} \beta.$$

A permanent increase in benefits. The calibration, instead, uses as a target the micro-elasticity of unemployment duration with respect to a rise in benefits that is anticipated to prevail during the household's entire unemployment spell. That is, a rise in benefits that also affects the continuation value. By (118), in steady state

$$\frac{\partial \log \Delta'}{\partial \log c'_u} = -1 - (1-\xi)\frac{\psi_s s}{1-s}\frac{\partial \log s'}{\partial \log c'_u} + \beta(1-\xi)E\frac{\partial \log \Delta'}{\partial \log c'_u}$$

From this, by (117)

$$\frac{\partial \log \Delta'}{\partial \log c'_u} = -1 - (1-\xi)sf\beta E\frac{\partial \log \Delta'}{\partial \log c'_u} + \beta(1-\xi)E\frac{\partial \log \Delta'}{\partial \log c'_u},$$

or, in steady state,

$$\frac{\partial \log \Delta'}{\partial \log c'_u} = -\frac{1}{1-\beta(1-\xi)(1-sf)}$$

With this, by (116) and(117)

$$\left.\frac{\partial \log D}{\partial \log c'_u}\right|_{\text{anticipated raises}} = \frac{f(1-s)}{\psi_s}\beta\frac{1}{1-\beta(1-\xi)(1-sf)}.$$

Or,

$$\left.\frac{\partial \log D_2}{\partial \log c'_u}\right|_{\text{anticipated raises}} = \epsilon_{D_2,b}\frac{1}{1-\beta(1-\xi)(1-sf)}.$$

In essence, the effect on the household's search behavior differs in the two cases. When benefits are anticipated to remain high, households anticipate that also the future surplus from search is diminished. They, thus, reduce their search effort by more. Note that for $\beta \approx 1$ and ξ small, $\frac{1}{1-\beta(1-\xi)(1-sf)} \approx \frac{1}{sf} = D$. Thus, in this case,

$$\left.\frac{\partial \log D_2}{\partial \log c'_u}\right|_{\text{anticipated raises}} = D \cdot \epsilon_{D_2,b},$$

exactly the product that appears in equation (32). Furthermore, in our calibration the duration of unemployment is long, so that $D \cdot \epsilon_{D_2,b} \approx \left.\frac{\partial \log D}{\partial c'_u}\right|_{\text{anticipated increase}}$, that is, approximately equal to the duration of unemployment with respect to benefits to which we calibrate.

C Finding optimal policies and welfare gains using perturbation

This section describes the algorithm that we use to derive the optimal federal RI scheme in Section 4.3. We wish to have closed-form expressions for the value of the objective function (for given member-state policies and a given federal RI scheme). When accounting for the transition, in Sections 4.3.1 and 4.3.3, we find policies that maximize the conditional expectation of the objective. In these cases, we condition on the initial state being the non-stochastic steady state implied by our calibrated model (Table 4). To obtain optimal federal RI 4.3.2, we find federal and member-state policies that maximize the unconditional mean of the objective function. The formulae for the first moments are given in Appendix D. Next, we describe how we search for the optimal federal RI scheme. We describe this for the case when we account for the transitions.

Finding the optimal federal RI scheme. The algorithm proceeds as follows.

1. The goal is to find values $\boldsymbol{\phi} = [\alpha, \delta]' \in \mathbb{R}^2$ and $\boldsymbol{\tau}_F$ that solve the federal government's problem (25) anticipating the member states' policy choices. The values of $\boldsymbol{\phi}$ induce payout function $\mathbf{B}_F(\,\cdot\,;\,\cdot\,)$.

2. Find $\boldsymbol{\phi}$ by numerical optimization. For this, for each try $\boldsymbol{\phi}$, evaluate the federal government's objective function using conditional expectations for a given initial state.

3. In order to evaluate the federal planner's objective function for given $\boldsymbol{\phi}$, make sure that the scheme is feasible in light of the member states' optimal response. In particular, a federal RI policy has to be self-financing in light of member states' responses; recall (23). For fixed ϕ we iterate as follows.

    (a) mark the iteration by $^{(n)}$. Set $n = 0$. Start from an initial value of $\boldsymbol{\tau}_F^{(-1)}$.

    (b) set $\boldsymbol{\tau}_F = \boldsymbol{\tau}_F^{(n-1)}$. For given federal RI policy $\boldsymbol{\phi}$ and $\boldsymbol{\tau}_F = \boldsymbol{\tau}_F^{(n-1)}$, let the member state solve (26).

    (c) label the maximizing member-state policies $\{\tau_v^{i,(n)}, \tau_\xi^{i,(n)}, b^{i,(n)}\}$. These induce a law of motion $\mu_0^{(n)}$ for the distribution and a value for the objective function of $\int W_0^{(n)} d\mu_0^{(n)}$.

    (d) for given member-state policies $\{\tau_v^{i,(n)}, \tau_\xi^{i,(n)}, b^{i,(n)}\}$, and given federal policy $\boldsymbol{\phi}$, find a value $\boldsymbol{\tau}_F^{(n)}$ that solves the federal RI scheme's financing constraint (23) for these given policies and given the induced dynamics for the member-state economies.

    (e) if $\boldsymbol{\tau}_F^{(n)}$ is not sufficiently close to $\boldsymbol{\tau}_F^{(n-1)}$, set $n = n + 1$ and go to step 3b. Else, set $\boldsymbol{\tau}_F = \boldsymbol{\tau}_F^{(n)}$ and go to step 4.

4. The federal policy implied by $\boldsymbol{\phi}$ and $\boldsymbol{\tau}_F$ is feasible. Set $\int W_0 d\mu_0 = \int W_0^{(n)} d\mu_0^{(n)}$.
5. Continue the numerical optimization started in step 2 until the maximum for the federal government's objective is found.

D Fourth-order-accurate first moments

In this section we consider a pruned perturbation solution to a dynamic stochastic general equilibrium (DSGE) model. We derive recursions for computing fourth-order-accurate unconditional first moments of the model's endogenous variables and for the conditional expected transition path, of which the period-0 entry is the conditional expectation. In terms of notation, we follow Levintal (2017) who provides up to fifth-order accurate approximations. Levintal (2017) in turn follows Andreasen et al. (2018) who provide up to third-order pruned solutions.

D.1 Preliminaries

We consider the following class of DSGE models. Let $y_t \in \mathbb{R}^{n_y}$ be a vector of control variables and $x_t \in \mathbb{R}^{n_x+1}$ a vector of state variables that includes a perturbation parameter $\sigma \geq 0$. Consider a perturbation solution to a DSGE model around the steady state $x_{SS} = 0$. The exact solution to the model is given by

$$\begin{aligned} y_t &= g(x_t), \\ x_{t+1} &= h(x_t) + \sigma\eta\epsilon_{t+1}, \end{aligned} \tag{119}$$

where ϵ_{t+1} follows an n_ϵ dimensional multivariate normal distribution and is independently and identically distributed in each period. Solving a DSGE model amounts to finding unknown functions g and h.
For most DSGE models, the full solution to system (119) cannot be found explicitly. The perturbation solution approximates the true solution using a Taylor series expansion around the steady state, $x_t = x_{t+1} = 0$. Up to fourth order, we have

$$x_{t+1} = h_x x_t + \frac{1}{2} h_{xx} x_t^{\otimes 2} + \frac{1}{6} h_{xxx} x_t^{\otimes 3} + \frac{1}{24} h_{xxxx} x_t^{\otimes 4} + \sigma\eta\varepsilon_{t+1}, \tag{120}$$

where $h_x, h_{xx}, \ldots$ denote first-, second-, etc., order derivatives of function h with respect to vector x. Superscript $^{\otimes n}$ represents the n-th Kronecker power, i.e., $x^{\otimes n} = \overbrace{x \otimes x \otimes \ldots}^{n \text{ times}}$.
However, the system (120) may display explosive dynamics and may not have any finite unconditional moments (Andreasen et al., 2018) . The solution to this problem is pruning the state space of the approximated solution so as to remove explosive paths. Following Levintal (2017), the pruned 4th-order approximation of the state variables reads

$$x_{t+1} = x_{t+1}^f + x_{t+1}^s + x_{t+1}^{rd} + x_{t+1}^{4th}, \tag{121}$$

where

$$x_{t+1}^f = h_x x_t^f + \sigma\eta\varepsilon_{t+1}, \tag{122}$$

$$x_{t+1}^s = h_x x_t^s + \frac{1}{2}h_{xx}\left(x_t^f\right)^{\otimes 2}, \tag{123}$$

$$x_{t+1}^{rd} = h_x x_t^{rd} + \frac{1}{2}h_{xx}\left(2\left(\left(x_t^f \otimes x_t^s\right)\right)\right) + \frac{1}{6}h_{xxx}\left(x_t^f\right)^{\otimes 3}, \tag{124}$$

and

$$x_{t+1}^{4th} = h_x x_t^{4th} + \frac{1}{2}h_{xx}\left(2\left(x_t^f \otimes x_t^{rd}\right) + \left(x_t^s\right)^{\otimes 2}\right) + \frac{1}{6}h_{xxx}\left(3\left(x_t^f\right)^{\otimes 2} \otimes x_t^s\right) + \frac{1}{24}h_{xxxx}\left(x_t^f\right)^{\otimes 4}. \tag{125}$$

And the fourth-order accurate, pruned solution for jump variables y_t is given by

$$y_t = y_t^f + y_t^s + y_t^{rd} + y_t^{4th}. \tag{126}$$

Here,

$$y_t^f = g_x x_t^f \tag{127}$$

$$y_t^s = g_x x_t^s + \frac{1}{2}g_{xx}\left(x_t^f\right)^{\otimes 2}, \tag{128}$$

$$y_t^{rd} = g_x x_t^{rd} + \frac{1}{2}\, g_{xx}\, 2\left(x_t^f \otimes x_t^s\right) + \frac{1}{6}g_{xxx}\left(x_t^f\right)^{\otimes 3}, \tag{129}$$

and

$$y_t^{4th} = g_x x_t^{4th} + \frac{1}{2}g_{xx}\left(2\left(x_t^f \otimes x_t^{rd}\right) + \left(x_t^s\right)^{\otimes 2}\right) + \frac{1}{6}g_{xxx}\left(3\left(x_t^f\right)^{\otimes 2} \otimes x_t^s\right) + \frac{1}{24}g_{xxxx}\left(x_t^f\right)^{\otimes 4}. \tag{130}$$

Note that if the shock is drawn from the standard normal distribution, as is the case in the model developed in the current paper, then

$$\begin{aligned}
&\mathrm{E}\left(\varepsilon_t\right)^{\otimes 2} = vec(I_{n_e}),\\
&\mathrm{E}\left(\varepsilon_t\right)^{\otimes 3} = 0, \text{and}\\
&\mathrm{E}\left(\varepsilon_t\right)^{\otimes 5} = 0.
\end{aligned}$$

Let $M^4 \equiv \mathrm{E}\left(\varepsilon_t\right)^{\otimes 4}$ be the kurtosis of the standard multivariate normal distribution.

D.2 Rules for Kronecker products

In the course of the proofs we will use extensively the following (well-known) properties of the Kronecker product.[26] These are:

$$\begin{aligned}
\mathbf{A} \otimes (\mathbf{B} + \mathbf{C}) &= \mathbf{A} \otimes \mathbf{B} + \mathbf{A} \otimes \mathbf{C}, \\
(\mathbf{A} + \mathbf{B}) \otimes \mathbf{C} &= \mathbf{A} \otimes \mathbf{C} + \mathbf{B} \otimes \mathbf{C}, \\
(k\mathbf{A}) \otimes \mathbf{B} &= \mathbf{A} \otimes (k\mathbf{B}) = k(\mathbf{A} \otimes \mathbf{B}), \\
(\mathbf{A} \otimes \mathbf{B}) \otimes \mathbf{C} &= \mathbf{A} \otimes (\mathbf{B} \otimes \mathbf{C}), \\
(\mathbf{AC}) \otimes (\mathbf{BD}) &= (\mathbf{A} \otimes \mathbf{B})(\mathbf{C} \otimes \mathbf{D}), \\
vec(ABC) &= (C' \otimes A)vec(B).
\end{aligned}$$

We say that matrix $K_{m,n}$ of size $mn \times mn$ is an *commutation matrix* if it has the following property: Let A be an $(m \times n)$ matrix and B a $(p \times q)$ matrix. Then

$$K_{m,p}\,(A \otimes B)\,K_{q,n} = B \otimes A.$$

That is, the commutation matrix reverses the order of Kronecker product. The commutation matrix $K_{m,n}$ can be defined explicitly as

$$K_{n,m} = \sum_{i=1}^{m} \sum_{j=1}^{n} \left((e_i^m (e_j^n)') \otimes (e_j^n (e_i^m)') \right),$$

where e_i^m is the ith unit column vector of order m. For any commutation matrix $K_{p,q} = K_{q,p}^{-1}$.

D.3 Dynamics of the expected transition path

This section provides recursions for fourth-order accurate approximations for conditional expectations of the form $\mathrm{E}_t\, x_{t+j},\ j \geq 1$. Our goal is to characterize the following expressions

$$\mathrm{E}_t\, x_{t+j} = \mathrm{E}_t\, x_{t+s}^{f} + \mathrm{E}_t\, x_{t+j}^{s} + \mathrm{E}_0\, x_{t+j}^{rd} + \mathrm{E}_0\, x_{t+j}^{4th},$$

where the decomposition follows from (121). We will look at each of the elements separately. And we constrain ourselves to Gaussian shocks. The derivations rely on using (122) through (130) and the rules for manipulating Kronecker products in Section D.2. These are straightforward manipulations. Here, we therefore only provide the results needed to implement the recursion. Armed with the transition dynamics for the state variables, the transition dynamics for the jump variables follow directly from (126) through (130).

[26] For example, Magnus, J. R. and Neudecker, H. (1999), Matrix Differential Calculus – with Applications in Statistics and Econometrics, Wiley.

D.3.1 First-order expected transition

From (122) and using that the shocks are iid, the first-order terms of the expected transition path are given by

$$\mathrm{E}_t\, x^f_{t+j} = h_x\, \mathrm{E}_t\, x^f_{t+j-1},\ j \geq 1 \tag{131}$$

D.3.2 Second-order expected transition

From (123)

$$\mathrm{E}_t\, x^s_{t+j} = h_x\, \mathrm{E}_t\, x^s_{t+j-1} + \frac{1}{2} h_{xx}\, \mathrm{E}_t \left[{x^f_{t+j-1}}^{\otimes 2} \right].$$

Here, using (122) and the fact that the shocks are iid zero mean, we have

$$\mathrm{E}_t \left[{x^f_{t+j}}^{\otimes 2} \right] = {h_x}^{\otimes 2}\, \mathrm{E}_t \left[{x^f_{t+j-1}}^{\otimes 2} \right] + (\sigma\eta)^{\otimes 2} vec(I_{n_e^2}).$$

D.3.3 Third-order expected transition

From (129),

$$\mathrm{E}_t\, x^{rd}_{t+j} = h_x\, \mathrm{E}_t\, x^{rd}_{t+j-1} + \frac{1}{2}\, h_{xx}\, 2\ \mathrm{E}_t \left[x^f_{t+j-1} \otimes x^s_{t+j-1} \right] + \frac{1}{6} h_{xxx}\, \mathrm{E}_t \left[{x^f_{t+j-1}}^{\otimes 3} \right].$$

Where

$$\mathrm{E}_t \left[x^f_{t+j} \otimes x^s_{t+j} \right] = h_x^{\otimes 2}\, \mathrm{E}_t \left[x^f_{t+j-1} \otimes x^s_{t+j-1} \right] + \frac{1}{2} \left[h_x \otimes h_{xx} \right] \mathrm{E}_t \left[{x^f_{t+j-1}}^{\otimes 3} \right],$$

In addition

$$\begin{aligned} \mathrm{E}_t \left[{x^f_{t+j}}^{\otimes 3} \right] \quad = \quad & h_x^{\otimes 3}\, \mathrm{E}_t \left[{x^f_{t+j-1}}^{\otimes 3} \right] \\ & \cdot \left[I_{n_x^3} + K_{n_x, n_x^2} + K'_{n_x, n_x^2} \right] \cdot \left[(\sigma\eta)^{\otimes 2} \otimes h_x \right] \cdot \left[vec(I_{n_e^2}) \cdot \mathrm{E}_t \left[x^f_{t+j-1} \right] \right]. \end{aligned}$$

D.3.4 Fourth-order expected transition

From (130),

$$\begin{aligned} \mathrm{E}_t\, x^{4th}_{t+j} \quad = \quad & h_x\, \mathrm{E}_t\, x^{4th}_{t+j-1} + \tfrac{1}{2} h_{xx} 2\, \mathrm{E}_t \left[x^f_{t+j-1} \otimes x^{rd}_{t+j-1} \right] \\ & + \tfrac{1}{2} h_{xx}\, \mathrm{E}_t \left[{x^s_{t+j-1}}^{\otimes 2} \right] + \tfrac{1}{6} h_{xxx} 3\, \mathrm{E}_t \left[{x^f_{t+j-1}}^{\otimes 2} \otimes x^s_{t+j-1} \right] + \tfrac{1}{24} h_{xxxx}\, \mathrm{E}_t \left[{x^f_{t+j-1}}^{\otimes 4} \right]. \end{aligned}$$

As to the respective terms,

$$\begin{aligned}\mathrm{E}_t\left[x^f_{t+j}\otimes x^{rd}_{t+j}\right] \quad = \quad & h_x^{\otimes 2}\,\mathrm{E}_t\left[x^f_{t+j-1}\otimes x^{rd}_{t+j-1}\right]\\ &+\left[h_x\otimes h_{xx}\right]\cdot \mathrm{E}_t\left[{x^f_{t+j-1}}^{\otimes 2}\otimes x^s_{t+j-1}\right]\\ &+\tfrac{1}{6}\left[h_x\otimes h_{xxx}\right]\cdot \mathrm{E}_t\left[{x^f_{t+j-1}}^{\otimes 4}\right].\end{aligned}$$

$$\begin{aligned}\mathrm{E}_t\left[{x^s_{t+j}}^{\otimes 2}\right] \quad = \quad & \left[h_x\otimes h_x\right]\cdot \mathrm{E}_t\left[{x^s_{t+j-1}}^{\otimes 2}\right]\\ &+\tfrac{1}{2}\left[I_{n_x^3}+K_{n_x,n_x^2}\right]\cdot\left[h_{xx}\otimes h_x\right]\cdot \mathrm{E}_t\left[{x^f_{t+j-1}}^{\otimes 2}\otimes x^s_{t+j-1}\right]\\ &+\tfrac{1}{4}\left[h_{xx}\otimes h_{xx}\right]\mathrm{E}_t\left[{x^f_{t+j-1}}^{\otimes 4}\right].\end{aligned}$$

$$\begin{aligned}\mathrm{E}_t\left[{x^f_{t+j}}^{\otimes 2}\otimes x^s_{t+j}\right] \quad = \quad & h_x^{\otimes 3}\cdot \mathrm{E}_t\left[{x^f_{t+j-1}}^{\otimes 2}\otimes x^s_{t+j-1}\right]\\ &+\left[(\sigma\eta)^{\otimes 2}\otimes h_x\right]\left[vec(I_{n_e^2})\otimes \mathrm{E}_t[x^s_{t+j-1}]\right]\\ &+\left[(\sigma\eta)^{\otimes 2}\otimes \tfrac{1}{2}h_{xx}\right]\cdot\left[vec(I_{n_e^2})\otimes \mathrm{E}_t[{x^f_{t+j-1}}^{\otimes 2}]\right]\\ &+\left[h_x^{\otimes 2}\otimes \tfrac{1}{2}h_{xx}\right]\cdot \mathrm{E}_t\left[{x^f_{t+j-1}}^{\otimes 4}\right].\end{aligned}$$

$$\begin{aligned}\mathrm{E}_t\left[{x^f_{t+j}}^{\otimes 4}\right] \quad = \quad & h_x^{\otimes 4}\cdot \mathrm{E}_t\left[{x^f_{t+j-1}}^{\otimes 4}\right]\\ &+\left[I_{n_x^4}+(K_{n_x,n_x}\otimes I_{n_x^2})\cdot\left[K'_{n_x^3,n_x}+K_{n_x^3,n_x}\right]+K'_{n_x^3,n_x}+K_{n_x^3,n_x}+K_{n_x^2,n_x^2}\right]\\ &\cdot\Big(\left[(\sigma\eta)^{\otimes 2}\cdot vec(I_{n_e})\right]\otimes\left[h_x^{\otimes 2}\cdot \mathrm{E}_t\,{x^f_{t+j-1}}^{\otimes 2}\right]\Big)\\ &+(\sigma\eta)^{\otimes 4}\,\mathrm{E}_t\,\varepsilon^{\otimes 4}_{t+j}.\end{aligned}$$

D.4 Analytical expressions for unconditional first moments

We seek to characterize the following expression

$$\mathrm{E}\,x_t=\mathrm{E}\,x^f_t+\mathrm{E}\,x^s_t+\mathrm{E}\,x^{rd}_t+\mathrm{E}\,x^{4th}_t,$$

Andreasen et al. (2018) derive the first three components, namely,

$$\mathrm{E}_0\,x^f_t=0$$

$$\begin{aligned}\mathrm{E}\,x^s_t \quad = \quad & (I_{n_x}-h_x)^{-1}\left[\tfrac{1}{2}h_{xx}(I_{n_x^2}-h_x\otimes h_x)^{-1}(\sigma^2\eta\otimes\eta)vec(I))+\tfrac{1}{2}h_{\sigma\sigma}\sigma^2\right]\\ & \mathrm{E}_0\,x^{rd}_t=0.\end{aligned}$$

We have that the fourth-order terms of the expecation are

$$
\begin{aligned}
\mathrm{E}\, x_t^{4th} = (I_{n_x} - h_x)^{-1} \Bigg[& \frac{1}{2} h_{xx} \,\mathrm{E} \left(2 \left(x_t^f \otimes x_t^{rd} \right) + (x_t^s)^{\otimes 2} \right) \\
& + \frac{1}{6} h_{xxx} \,\mathrm{E} \left(3 \left(x_t^f \right)^{\otimes 2} \otimes x_t^s \right) + \frac{1}{24} h_{xxxx} \,\mathrm{E} \left(x^f \right)^{\otimes 4} \\
& + \frac{3}{6} h_{\sigma\sigma x} \sigma^2 \,\mathrm{E}\, x_t^s + 6 \cdot \frac{1}{24} h_{\sigma\sigma xx} \sigma^2 \,\mathrm{E} \left(x_t^f \right)^{\otimes 2} + \frac{1}{24} h_{\sigma\sigma\sigma\sigma} \sigma^4 \Bigg].
\end{aligned}
$$

Where

$$
\begin{aligned}
\mathrm{E} \left(x_t^f \right)^{\otimes 4} = \sigma^2 \left(I_{n_x^4} - h_x^{\otimes 4} \right)^{-1} \Bigg[& \sigma^2 \eta^{\otimes 4} M^4 + \Bigg((h_x^{\otimes 2} \otimes \eta^{\otimes 2}) K_{n_e^2, n_x^2} \\
& + (h_x \otimes \eta \otimes h_x \otimes \eta) \left(I_{n_x} \otimes K_{n_x, n_e} \otimes I_{n_e} \right) K_{n_e^2, n_x^2} \\
& + (h_x \otimes \eta \otimes \eta \otimes h_x) \left(I_{n_x} \otimes K_{n_x, n_e^2} \right) K_{n_e^2, n_x^2} \\
& + (\eta \otimes h_x \otimes h_x \otimes \eta) \left(I_{n_e} \otimes K_{n_e, n_x^2} \right) \\
& + (\eta \otimes h_x \otimes \eta \otimes h_x) \left(I_{n_e} \otimes K_{n_e, n_x} \otimes I_{n_x} \right) \\
& + \left(\eta^{\otimes 2} \otimes h_x^{\otimes 2} \right) \Bigg) \left(vec(I_{n_e}) \otimes \mathrm{E}(x_t^f)^{\otimes 2} \right) \Bigg],
\end{aligned}
$$

$$
\mathrm{E} \left(x_t^f \right)^{\otimes 2} = \sigma^2 (I - h_x^{\otimes 2})^{-1} (\eta^{\otimes 2}) vec(I_{n_e})),
$$

$$
\begin{aligned}
\mathrm{E} \left[(x_t^s)^{\otimes 2} \right] = \left(I_{n_x^2} - h_x^{\otimes 2} \right)^{-1} \Bigg(& + .5 \left(K_{n_x, n_x} + I_{n_x^2} \right) (h_{xx} \otimes h_x) \,\mathrm{E} \left(\left(x_t^f \right)^{\otimes 2} \otimes x_t^s \right) + \frac{1}{4} h_{xx}^{\otimes 2} \,\mathrm{E} (x_t^f)^{\otimes 4} \\
& + \left(K_{n_x, n_x} + I_{n_x^2} \right) \left[\frac{1}{2} [(h_x \otimes h_{\sigma\sigma})(\sigma^2 x_t^s)] + \frac{1}{4} (h_{xx} \otimes h_{\sigma\sigma})(\sigma^2 (x_t^f)^{\otimes 2}) \right] + \frac{1}{4} h_{\sigma\sigma} \otimes h_{\sigma\sigma} \sigma^4 \Bigg),
\end{aligned}
$$

$$
\begin{aligned}
\mathrm{E} \left[\left(x_t^f \right)^{\otimes 2} \otimes x_t^s \right] = \left(I_{n_x^3} - h_x^{\otimes 3} \right)^{-1} \Bigg(& (\sigma^2 \eta^{\otimes 2} \otimes h_x) (\mathrm{E} \left[\epsilon_{t+1}^{\otimes 2} \right] \otimes \mathrm{E}\, x_t^s) + \frac{1}{2} (h_x^{\otimes 2} \otimes h_{xx}) \,\mathrm{E} (x_t^f)^{\otimes 4} \\
& + \frac{1}{2} (\sigma^2 \eta^{\otimes 2} \otimes h_{xx}) (vec(I_{n_e}) \otimes \mathrm{E} (x_t^f)^{\otimes 2}) + \frac{1}{2} (h_x^{\otimes 2} \otimes h_{\sigma\sigma}) \sigma^2 \,\mathrm{E} \left[(x_t^f)^{\otimes 2} \right] \\
& + \frac{1}{2} (\eta^{\otimes 2} \otimes h_{\sigma\sigma}) \sigma^4 \,\mathrm{E}\, \epsilon_{t+1}^{\otimes 2} \Bigg),
\end{aligned}
$$

and

$$
\begin{aligned}
\mathrm{E} \left[x_t^f \otimes x_t^{rd} \right] = \left(I_{n_x^2} - h_x^{\otimes 2} \right)^{-1} \Bigg(& (h_x \otimes h_{xx}) \,\mathrm{E} ((x_t^f)^{\otimes 2} \otimes x_t^s) + \frac{1}{6} (h_x \otimes h_{xxx}) \,\mathrm{E} \left(x_t^f \right)^{\otimes 4} \\
& + \frac{3}{6} (h_x \otimes h_{\sigma\sigma}) \sigma^2 \,\mathrm{E} \left[(x_t^f)^{\otimes 2} \right] \Bigg).
\end{aligned}
$$

E Finding optimal policies and welfare gains – global solution

This appendix describes the algorithm that we use in Section 4.5 of the main text to derive the optimal federal RI scheme with a threshold. We use a piece-wise linear interpolation scheme on a grid of points for the number of unemployed $u_t \in U$ and the number of average unemployed $u_t^{avg} \in U$. For U we use an equally-spaced grid between .03 and .16 with spacing of .005 and - to cover the extremes - add four additional equally-spaced points [.18;.2;.22;.24] of exceptionally high unemployment. This choice of grid ensures that we will not need any extrapolation. We use a discretized set of points to approximate the exogenous AR(1) process for productivity using 5 points. We then solve the model on a grid of points for the policy choices: For the underlying game we use an equally spaced grid of benefits b between 95% and 115% of the autarky steady state (with a grid space of .0025) to allow the individual country to optimize over. Next, we set an equally-spaced grid on the European refinancing tax τ_f between -.01 and .03 (with a grid space of .005) which we later use to solve for the European balanced budget. Last, we use an equally-spaced grid for the generosity of the European payout scheme, setting α between 0 (autarky) and 2 (with a grid space of .25). We check ex-post that the boundaries of the grids above are not played at the optimal solution, so that extrapolation is not necessary to compute the equilibrium. We choose an indicator variable such that payouts or taxes are paid only if $|u_t - u_t^{avg}| \geq \Phi$, where we use as a benchmark that $\Phi = .015$. This makes the European scheme (22) non-linear[27]

$$\mathbf{B}_F(u_t - u_t^{avg}) = \alpha(u_t - u_t^{avg}) I(|u_t - u_t^{avg}| \geq \Phi = .015). \tag{132}$$

For given parameters (including fixing the hiring subsidies and separation taxes at their autarkic steady-state values), in an inner loop, we first solve for the market equilibrium in the individual country; for a given federal scheme and a given replacement rate b:

Inner Loop

1. At stage 0, guess choices for search s_0, separations ξ_0, job-finding rates f_0, wages w_0 and consumption $c_{e,0}$ as well as initial guesses for values J_0 and utility differences Δ_0 as a function of the states. For given replacement rates and European policy parameters we can calculate $c_{u,0}$ and dividends.

2. In each iteration j, given guessed policy functions, we obtain tomorrow's states u' and u'_{avg} using the laws of motion (equations (12) and (18)). Interpolate to obtain the discount kernel Q'_j and calculate expected values EJ'_j and $E\Delta'_j$ using the discretized exogenous shock.

3. Use the value equations (equations (3) and (6) and (9)) to obtain updates on the candidate value functions J_j and Δ_j .

[27] We also looked at a one-sided rule where the European scheme only uses a constant refinancing tax in good times and a payout scheme that pays out only in bad times $\mathbf{B}_F(u_t - u_t^{avg}) = \alpha(u_t - u_t^{avg})\mathbb{I}(u_t - u_t^{avg} \geq \Phi = .015)$. This was inferior in terms of the welfare gain from federal RI.

4. Given these expected values of the firm and utility differences, use the first-order condition on search (equations (4) and (5)) to solve for a new candidate policy $s_{candidate}$. Use the optimal decision on separations (equation (8) and (17)) to update $\xi_{candidate}$. Use the free-entry condition (equation (10)) to update $f_{candidate}$ and rearrange the Nash-bargaining equation (equation (16)) by solving the firm value (evaluated at the old policies) for wages $w_{candidate}$. The matching function then delivers vacancies $v_{candidate}$.

5. Use the aggregate resource constraint, equation (19), to solve for an update on consumption $c_{e,j}$ and ensure feasibility.

6. Choose a gradual updating scheme that is parameterized separately for each endogenous choice variable. For example, update the search choice according to $s_j = \varpi(s)s_{candidate} + (1 - \varpi(s))s_{j-1}$, where $\varpi(s)$ is the step size for the update of the search choice.

7. Iterate until for all choices, the distance d in the update step, i.e. $d(s_j, s_{j-1})$, is smaller than some pre-specified error.

Outer Loop (Game)
Given the policy functions from the inner loop, we next calculate welfare on all grid points. We interpolate welfare on a dense grid of points for any combination of European payouts α, the replacement rate b, and the refinancing tax $\boldsymbol{\tau}_F$ on their respective grids.
Next, for each α, we look for a fixed point in b and $\boldsymbol{\tau}_F$, such that b is the member state's best response (on the grid) in terms of welfare for a given α and $\boldsymbol{\tau}_F$ and that given the response b the federal scheme (for the given α) is self-financing. Note that, in order to look at self-financing schemes, $\boldsymbol{\tau}_F$ needs to be interpolated between the grid points for federal taxes.
This gives a value of welfare for each α on the grid. We then take the maximum over α of the welfare numbers. This gives the optimal European unemployment-based reinsurance scheme, with α on the grid, b on the grid and $\boldsymbol{\tau}_F$ interpolated between grid points.