Customer Acquisition, Business Dynamism and Aggregate Growth*

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Abstract

Business dynamism – the process of firm entry, growth and exit – lies at the heart of modern endogenous growth models. While productivity differences have traditionally been seen as the main driving forces of business dynamism, a growing body of evidence suggests that customer acquisition frictions are at least as important. In light of this evidence, we propose a novel endogenous growth model in which innovating firms must first acquire customers to sell their products. Estimating our model with aggregate and firm-level data, we find that expansions of firms' customer bases (market sizes) boost their incentives to innovate and shift resources towards high-growth businesses ("gazelles"). Combined, these effects explain over 1/3 of aggregate growth and substantially change predictions about the efficacy of growth policies. Finally, we document support for key model predictions using firm-level micro-data.

Keywords: Customer acquisition, Firm Heterogeneity, Growth, Innovation, R&D

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1 Introduction

The continuous process of firm entry, growth and exit – "business dynamism" – plays a key role in aggregate job creation and productivity growth (Haltiwanger, 2012). By now, models incorporating business dynamism have been employed to understand a range of important questions regarding e.g. international trade, long-run growth, unemployment, inequality, market power, and monetary and fiscal policy.[1] Typically, such models think of business dynamism as a purely productivity-driven phenomenon whereby less productive firms shrink and shut down, giving way to more productive businesses.

However, a growing body of recent empirical and theoretical research stresses that business dynamism is in fact largely driven by customer accumulation.[2] In this paper, we argue that firm-level customer acquisition is also a quantitatively important driver of aggregate economic growth. Moreover, we show that ignoring customer accumulation leads to substantially different conclusions about the efficacy of growth policies. We then proceed to document empirical support for key theoretical predictions of our framework in firm-level micro-data.

To make our point, we propose a new model of endogenous growth which explicitly considers firm-level customer accumulation. In our framework – as in existing growth models – firms invest into improving their individual productivity levels, the ultimate source of aggregate growth. In contrast to existing growth models, however, in our framework firms must first acquire customers in order to sell their products. This creates a novel link between innovation and customer accumulation which, as we document, has important macroeconomic consequences.

We first illustrate the key mechanism by considering a simplified – "benchmark" – version of our model in which customer acquisition is exogenous. This assumption facilitates analytical characterizations and allows us to build intuition. As we document later, this intuition caries over to our full model in which we micro-found and endogenize customer acquisition.

In the benchmark model, we show three central theoretical results. First, customer acquisition increases firms' incentives to conduct R&D through a "firm-level market size effect". To understand this, note that higher firm-level productivity allows businesses to sell at lower prices and earn higher profits. Acquiring new customers, therefore, raises the benefits of innovation because the resulting lower prices can apply to a larger market.[3]

[1] See e.g. Melitz (2003), Klette and Kortum (2004), Kaas and Kircher (2015), Jones and Kim (2018), Peters (2020), Ottonello and Winberry (2020), Spencer (2022).

[2] See e.g. Foster et al. (2008), Dinlersoz and Yorukoglu (2012), Foster et al. (2016), Hottman et al. (2016), Kaas and Kimasa (2021), Bernard et al. (2022), Eslava and Haltiwanger (2021), Einav et al. (2022), Rudanko (2022).

[3] A range of studies have documented the presence of firm-level market size effects in the data (see e.g. Acemoglu and Linn, 2004; Jaravel, 2019; Aghion et al., 2020). See Schmookler (1966) and Scherer (1982) for early contributions on "demand-pull" and technological innovation.

Second, aggregating firm-level innovation decisions leads to our next result: the presence of customer acquisition at the firm-level raises *aggregate* economic growth. It is important to note that customer acquisition raises aggregate growth only indirectly by boosting firms' incentives to innovate – our first theoretical result. Indeed, the aggregate customer base is fixed in our framework.

Third, customer acquisition changes firms' responsiveness to R&D subsidies. Specifically, the prospect of larger market shares associated with customer acquisition tilts firms' discounting towards future periods. This makes businesses more sensitive to future declines in innovation costs brought about by R&D subsidies.

To quantify the importance of our theoretical results, we first generalize the benchmark model along several dimensions – most importantly by considering endogenous customer acquisition. We do so by allowing firms to use prices in order to attract more customers along the lines of e.g. Drozd and Nosal (2012), Gourio and Rudanko (2014), Foster et al. (2016) and Rudanko (2022).[4] In addition, Appendix D.5 extends our model to also allow firms to accumulate customers via advertising. Importantly, we show that this does not affect our price-driven channel and only strengthens endogenous customer accumulation – the key novel channel in our framework.

The combination of endogenous innovation and customer acquisition gives rise to a novel feedback loop between firms' productivity levels and their customer bases. In particular, since more productive businesses can produce demanded goods at lower costs, they can invest more into customer acquisition. At the same time, however, the associated increase in firms' customer bases (market sizes) boosts their incentives to conduct R&D – the firm-level market size effect described above. This endogenous feedback loop not only strengthens returns to innovation, but also shifts market shares and resources towards businesses with high-productivity growth – often referred to as "gazelles" in the literature.[5] Our framework, therefore, offers a new underpinning for the documented macroeconomic importance of high-growth firms (see e.g. Haltiwanger et al., 2016).

To isolate the effect of customer acquisition on firm-level and aggregate outcomes, we consider *counterfactual* firm-level decisions. These are based on firms assuming that their respective customer bases are fixed. The contribution of customer acquisition is then quantified by the difference between a particular outcome in the generalized model and its counterfactual analogue.

Estimating our model with indirect inference on a combination of aggregate and firm-level data, we show that customer acquisition accounts for about 20 percent of firms'

[4] Examples underlying this modeling approach include not only initial discount periods on products – such as newspaper subscriptions, mobile phone plans or gym memberships – but also longer-term pricing strategies to build sufficiently large customer bases – including those applied by service and technology firms such as AirBnB, DoorDash, Netflix or Uber.

[5] This effect is similar in spirit, but distinct from, the "reallocation effect" identified in existing growth models (see e.g. Lentz and Mortensen, 2005). In our framework, reallocation is partly driven by customer acquisition and, therefore, occurs even in the absence of differences in firm-level productivity.

innovation on average. This contribution is more than twice as high (42 percent) among young firms which enjoy the strongest customer growth on average. At the aggregate level, the model suggests that more than 1/3 of aggregate growth is accounted for by customer acquisition. This overall impact is a combination of two forces. First, the impact of customer accumulation on firms' innovation incentives – as shown analytically and discussed above. Second, the endogenous feedback loop between firm-level productivity and customer acquisition which shifts resources and market shares towards gazelles.

As a final step in our quantitative analysis, we turn our attention to analyzing the impact of two common policy interventions: subsidies to R&D and firms' operations. We do so in our generalized model and an identical economy which, however, ignores customer accumulation.[6] First, we show that in the presence of customer acquisition R&D subsidies are much more successful at boosting aggregate growth – confirming our analytical results.

In contrast, we document that subsidizing firms' operations has a much smaller growth effect in the generalized model compared to an economy which ignores customer accumulation. The key reason is the different nature of business dynamism across our two economies. In an economy without customer accumulation, firm growth and survival are purely productivity-driven. In the generalized model, however, firms can survive by being productive and/or by enjoying a large customer base. Therefore, compared to a model without customer acquisition, in our framework subsidizing firms' operations has a less direct impact on growth. This is because the overall effect depends on changes to the *joint* distribution of firm-level productivities and customer bases.

Despite focusing only on two policy examples, our results are more general. This is because our conclusions ultimately depend on how customer accumulation shapes business dynamism and, in turn, aggregate outcomes. Therefore, using a model without customer accumulation to analyze any policies (or shocks) which affect business dynamism may provide a skewed view of their aggregate impact.

Finally, to establish the empirical relevance of the proposed mechanisms, we turn to firm-level data and document support for key model predictions. First, we use variation across industries to show a positive link between firms' customer accumulation and their R&D expenditures. Second, we document that R&D subsidies have heterogeneous firm-level effects consistent with our model predictions. To establish these results, we use variation in the extent and inception date of R&D incentives across U.S. states, collected by Wilson (2009).

Our paper straddles two large bodies of research. First, we build on the literature studying firm dynamics (see e.g. Jovanovic, 1982; Hopenhayn and Rogerson, 1993, for seminal contributions) and in particular on papers analyzing demand-side factors and cus-

[6] We do not strive to determine optimal policies. Instead, we illustrate quantitatively that commonly used policies have a vastly different impact in environments with and without customer accumulation.

tomer acquisition (see e.g. Drozd and Nosal, 2012; Dinlersoz and Yorukoglu, 2012; Gourio and Rudanko, 2014; Arkolakis, 2016; Perla, 2019; Kaas and Kimasa, 2021; Rudanko, 2022).[7] We add to these papers the context of aggregate growth. Second, our paper connects to endogenous growth models which, however, typically ignore demand-side factors (see e.g. Klette and Kortum, 2004; Lentz and Mortensen, 2008; Akcigit and Kerr, 2018; Acemoglu et al., 2018). Exceptions include Cavenaile and Roldan-Blanco (2021) and Einav et al. (2022). The former present a model in which advertising statically increases (perceived) product quality. The latter use Visa transaction data to document the importance of customers for sales (growth) heterogeneity in the retail sector and rationalize these findings using a model in which customers are a static function of current advertising expenditures.[8] While in both these models advertising affects sales in a static manner, our paper stresses the dynamic nature of firm-level customer accumulation and how it forms an endogenous feedback loop with firm-level productivity growth.

The paper is structured as follows. The next Section describes the benchmark framework and derives our central analytical results. Sections 3, 4 and 5, respectively, describe the generalized model, parameterize it and provide quantitative results. Section 6 documents support for key model predictions and the final section concludes.

2 A benchmark theoretical framework

The primary goal of our theoretical analysis is to study how customer acquisition and R&D investment influence each other at the firm level and, in turn, how their interaction affects aggregate growth. We begin by considering a relatively restricted environment which, however, allows us to derive closed-form solutions and build intuition that – as will become clear – carries over to our full model. In particular, we start by considering a "benchmark" model with *exogenous* customer acquisition. In the next section, we endogenize customer accumulation and extend the benchmark along several other dimensions.

2.1 The model

In what follows we use upper-case letters to denote aggregates and lower-case letters to denote firm-level variables. Time is discrete and we use primes to indicate next period's values.

[7] Dynamic customer accumulation has also been studied in various other macroeconomic contexts not directly related to business dynamism, see e.g. Ravn et al. (2006), Nakamura and Steinsson (2011), Petrosky-Nadeau and Wasmer (2015), Shi (2016) or Fernández-Villaverde et al. (2021).

[8] A related paper is Rachel (2021), which studies leisure-enhancing technologies (and associated advertising) and how they raise firms' sales via brand equity.

Consumers: Preferences. We assume a representative household which consists of workers who supply labor, N, consume a final good, C, and invest into assets, A.[9] Aggregate consumption – the economy's numeraire with a normalized aggregate price $P = 1$ – is a CES aggregation of individual goods varieties, c_j:

$$C = \left[\int_{j\in\Omega} d_j^{\frac{1}{\eta}} c_j^{\frac{\eta-1}{\eta}} dj \right]^{\frac{\eta}{\eta-1}}, \tag{1}$$

where Ω is the and positive mass of producers and where $\eta > 1$ is the elasticity of substitution between goods varieties.[10]

A key novelty of our framework lies in the treatment of $d_j > 0$, the utility weights or "demand stocks" of individual varieties. While existing endogenous growth models typically treat d_j as constant and common across firms, we allow them to vary over time. More specifically, as explained below, we allow firms to invest into expanding the demand stock for their respective products. While not explicitly modelled here, this formulation can be micro-founded via firms' investment into making household members aware of their products (see e.g. Dinlersoz and Yorukoglu, 2012; Sedláček and Sterk, 2017).

Consumers: Optimal decisions. The representative household maximizes life-time utility, $\mathbb{U} = \sum_{t=0}^{\infty} \beta^t (\ln C_t - \upsilon N_t)$, subject to a budget constraint

$$A' + \int_{j\in\Omega} p_j c_j = WN + (1+R)A, \tag{2}$$

where β is the discount factor, υ is the disutility of labor, total assets are given by $A = \int_{j\in\Omega} v_j dj$, with v_j being the value of a firm producing variety j, p_j are variety-specific prices (relative to the aggregate price index P), W is the wage and R is the interest rate. The resulting Euler equation, labor supply and demand for individual goods take on familiar forms:

$$1 = \beta \mathbb{E} \frac{C}{C'} (1 + R'), \tag{3}$$

$$W = \upsilon C, \tag{4}$$

$$c_j = d_j p_j^{-\eta} C. \tag{5}$$

[9] For tractability, we abstract from workers' skills. For an analysis of how labor heterogeneity impacts growth, see e.g. Acemoglu et al. (2018).

[10] We choose the CES structure for direct comparability with the majority of existing models of endogenous growth (see e.g. Acemoglu et al., 2018). We discuss the implications of this choice for our results in Section 5. With slight abuse of notation, Ω refers both to the mass and set of produces.

Firms: Production and R&D. Each goods variety is produced by an individual firm using labor, n_j, and a firm-specific (endogenous) productivity level, q_j, according to:

$$c_j = q_j n_j. \tag{6}$$

Firms can strive to improve their production efficiency by investing s_j units of labor into R&D. In return, they obtain an innovation probability x_j determining the success rate of their R&D efforts. We assume that R&D costs are convex in the innovation probability:

$$s_j = \overline{s}_j x_j^{\psi}, \tag{7}$$

where $\psi > 1$ and $\overline{s}_j > 0$ is a potentially firm-specific scaling factor described below. If successful, innovations lead to an increase in firm-level productivity by a factor of $(1+\lambda)$, where $\lambda > 0$:

$$q_j' = \begin{cases} q_j(1+\lambda) & \text{with probability } x_j \\ q_j & \text{with probability } 1 - x_j \end{cases} \tag{8}$$

Firms: Customer acquisition. As mentioned above, we allow firms to invest into their demand stock, d_j. Specifically, we assume that a firms' demand stock depends on the mass of accumulated customers, or "customer base", b_j:

$$d_j = b_j^{\gamma}, \tag{9}$$

where $\gamma > 0$ is a "pass-through" parameter from customers to demand. A firm's customer base evolves according to the following law of motion:

$$b_j' = (1-\zeta)(b_j + \Delta_j), \tag{10}$$

where ζ is a customer depreciation rate and Δ_j is a firm-specific customer acquisition factor which potentially varies over firms' life-cycles. At this stage, we treat Δ_j and its evolution – described in detail below – as exogenous. The next section then endogenizes Δ_j and the resulting accumulation of customers.

Firms: Entry and exit. In the benchmark model, we assume that incumbent businesses shut down at an exogenous rate, δ, and are replaced by an equal mass, $\delta\Omega$, of entrants. All startups are assumed to enter with a common initial stock of customers normalized to $b_e = 1$, and to obtain a random draw of initial firm-specific productivity, q_e. The latter is distributed identically and independently across firms according to a cumulative distribution function H_e.[11] Thereafter, startups behave as incumbent firms

[11] Initial production efficiency, q_e, is assumed to be proportional to last period's aggregate productivity index (Q_{-1}) defined below. This setup is characterized by entrants "standing on the shoulders of giants"

described above.

Firm entry and exit introduces a degree of churn into demand stocks as mature exiting firms – with higher levels of accumulated customers – are replaced by startups facing a low initial customer base, b_e. As we show below, even with exogenous customer accumulation this churn can ensure the stationarity of the aggregate customer base, $B = \int_j b_j dj$.

Firms: Optimal decisions. Firms choose prices, production labor and R&D investment in order to maximize firm value:

$$v(q_j, b_j) = \max_{p_j, n_j, s_j} \left\{ \begin{array}{c} p_j c_j - W(n_j + s_j) + \\ \\ \frac{1-\delta}{1+R'} \left[\begin{array}{c} x_j v\left(q_j(1+\lambda), b_j'\right) \\ \\ +(1-x_j) v_j \left(q_j, b_j'\right) \end{array} \right] \end{array} \right\}, \tag{11}$$

$$\text{s.t. } c_j = b_j^{\gamma} p_j^{-\eta} C, \quad c_j = q_j n_j, \quad s_j = \overline{s}_j x_j^{\psi}, \quad x_j \in [0,1],$$

where we have made explicit the dependence of firm value on levels of productivity and the customer base. Optimal pricing and innovation conditions can be written as:

$$p_j = \frac{\eta}{\eta - 1} \frac{W}{q_j}, \tag{12}$$

$$\psi W \frac{s_j}{x_j} = \frac{1-\delta}{1+R'} \left[v\left(q_j(1+\lambda), b_j'\right) - v\left(q_j, b_j'\right) \right], \tag{13}$$

where prices are set as a constant markup over marginal costs (12) and the optimal innovation investment is a balance between marginal costs and benefits (13). The latter depends on the change in firm value brought about by a successful innovation – the term in square brackets on the right-hand-side of (13). However, in contrast to existing models, in our framework R&D decisions now also depend on the future customer base, b_j'.

Market clearing and aggregate growth. Labor market clearing requires that all labor demanded by firms (for production and R&D) is supplied by the household

$$N = \int_{j \in \Omega} (n_j + s_j) dj. \tag{14}$$

We focus on the balanced growth path (BGP) of the economy, along which all growing variables grow at the same rate $1 + g = Q'/Q$, where Q is the aggregate productivity

as startups inherit productivity advances from past incumbents (see e.g. Akcigit and Kerr, 2018).

index:

$$Q \equiv \left(\int_{j\in\Omega} d_j q_j^{\eta-1} dj \right)^{\frac{1}{\eta-1}}. \tag{15}$$

Intuitively, our economy grows at the pace of *demand weighted* average firm-level productivity growth (adjusted for the elasticity of substitution in consumption). Therefore, along the BGP we can stationarize our economy by dividing all growing variables by Q. In what follows, we denote stationarized variables with "hats", e.g. $\hat{C} = C/Q$. Appendix A presents a formal definition of the balanced growth path equilibrium.

2.2 Central theoretical results

This section presents three sets of analytical results stemming from our benchmark model. The three propositions shed light on how the presence of firm-level customer accumulation alters firms' incentives to conduct R&D, how it impacts aggregate growth and how it changes the economy's sensitivity to R&D subsidies.[12]

Parametric and functional form assumptions. The following assumptions – which we relax in the next section – allow us to derive closed form solutions.

Assumption **1** (Parameter restrictions and functional forms)
Assume the following hold:
(a) Parameter restrictions: $\eta = \psi = 2$, $\gamma = 1$ *and* $0 < \zeta < \delta$,
(b) Customer base accumulation: $\Delta_j = \overline{\Delta}/b_j$ *for all* j *and in every period, with* $\overline{\Delta} \in (\frac{\zeta}{1-\zeta}, \frac{\delta}{1-\delta})$,
(c) R&D scaling: $\overline{s}_j = \overline{s} b_j \hat{q}_j^{\eta-1}$, *with* $\overline{s} > 0$.

Part (a) of Assumption 1 is not particularly restrictive. Empirical evidence on the curvature of the R&D cost function typically points to a value of $\psi = 2$ (see e.g. Hall et al., 2001; Bloom et al., 2002) and assuming $\eta = 2$ and $\gamma = 1$ is done without loss of generality. Part (b), together with the restriction on customer depreciation ζ, implies a life-cycle profile of Δ_j such that the customer base grows at a positive, constant rate common to all firms, $1 + g_b = (1-\zeta)(1+\overline{\Delta}) > 1$ and that the aggregate customer base, B, is constant.[13] Finally, Part (c) restricts R&D scaling in a way that renders firm values proportional to their market shares.

[12] We defer all proofs to Appendix A where we also present several other analytical results, such as the model's predictions about firm size growth and Gibrat's law.

[13] Because of common customer base growth, g_b, and initial conditions, b_e, all businesses of the same age have the same customer base, b_a. Using μ_a to denote the mass of firms of particular age a (with the mass of startups being $\mu_0 = \delta\Omega$) and restricting $g_b \in (\frac{\zeta}{1-\zeta}, \frac{\delta}{1-\delta})$, we can write $B = \int_{j\in\Omega} b_j dj = \sum_{a=0}^{\infty} b_a \mu_a = \sum_{a=0}^{\infty} (1+g_b)^a b_e (1-\delta)^a \mu_0 = \frac{b_e \delta\Omega}{1-(1-\delta)(1+g_b)}$.

Customer acquisition, firm-level innovation and aggregate growth. We now present theoretical predictions regarding the influence of customer accumulation on firm-level and aggregate innovation.

Proposition **1** (Firm-level innovation)
(a) The firm-specific innovation rate $x \in [0, 1]$ *is implicitly given by*

$$x = \frac{\beta(1-\delta)}{\bar{s}(1+g)}(1+g_b)\lambda\mathcal{A}(x),$$

where $\mathcal{A}(x) > 0$.
(b) Firm-level innovation increases with customer base growth

$$\frac{\partial x}{\partial g_b} > 0.$$

Part (a) of Proposition 1 shows that the optimal innovation rate is constant and independent of production efficiency, q, and the demand stock, d. Therefore, it is common across all businesses and thus independent of firm size as in other existing growth models (see e.g. Klette and Kortum, 2004). Despite the innovation rate being independent of the *level* of a firm's customer base, Part (b) of Proposition 1 shows that it does depend on the speed of customer base accumulation, g_b.

To understand this further, recall that firms' R&D decisions are driven by expected future profits (13). These, in turn, depend on both future production efficiency and a firm's customer base. A successful innovation allows businesses to charge lower prices and, therefore, to raise sales as the household tilts more of its consumption towards their product. These benefits from innovation are magnified by customer accumulation because the resulting lower prices can be applied to a larger market.

The channel through which customer accumulation raises the incentives to conduct R&D is akin to the "market size effect" identified at the aggregate level in earlier vintages of endogenous growth models (see e.g. Jones, 1995). However, our theory predicts a *firm-level market size effect* – consistent with a range of empirical studies (see e.g. Acemoglu and Linn, 2004; Jaravel, 2019; Aghion et al., 2020).[14]

[14] This effect occurs because b and q are complements in the profit function, as in e.g. Einav et al. (2022). However, the contemporaneous cross-sectional correlation of firms' customer bases and productivity is less clear as it depends on how R&D costs scale with firm size (which we estimate to match the data in the generalized model). For instance, if R&D costs depended on the expected (instead of current) demand stock, innovation costs and benefits would scale identically and market size would not matter for firm-level innovation, see (13). While theoretically possible, this constitutes a knife-edge case which is in contrast to empirical evidence on firm-level market size effects. In addition, Cavenaile et al. (2021) provide evidence that advertising and R&D expenditures are substitutes in the cross-section of firms. Appendix D.5 extends our model to allow for advertising expenditures and shows that our model is in fact consistent with these findings.

Proposition **2** (Aggregate growth)
(a) Aggregate growth is given by

$$\frac{Q'}{Q} = 1 + g = 1 + \lambda x.$$

(b) Aggregate growth increases with firm-level customer base growth

$$\frac{\partial g}{\partial g_b} = \lambda \frac{\partial x}{\partial g_b} > 0.$$

(c) All else equal, let g^c be the counterfactual aggregate growth rate based on firms expecting no growth in customers, $g_b = 0$. Then, the impact of customer acquisition on aggregate growth has an upper bound given by

$$\frac{g - g^c}{g} = 1 - \frac{(1-\delta)(1-\beta)}{1-\beta(1-\delta)}.$$

Part (a) of Proposition 2 makes clear that aggregate economic growth is driven by firms' endogenous R&D choices. Notice that aggregate growth does not *directly* depend on the growth of firm-level demand stocks, reflecting our model's lack of aggregate market size effects discussed above.[15]

That said, Part (b) of Proposition 2 shows that expansions of the firm-level demand stock do impact aggregate economic growth *indirectly.* This is because firm-level customer base growth increases the incentives of individual firms to conduct R&D, raising their innovation rates (see Proposition 1).

Finally, Part (c) of Proposition 2 places a theoretical bound on the fraction of aggregate growth accounted for by the presence of customer acquisition. Employing empirically plausible annual values for the annual discount factor, $\beta = 0.97$, and average firm exit rates, $\delta = 0.08$, we obtain a back-of-the-envelope value of the contribution of customer acquisition to aggregate growth of almost 75%.

Therefore, Proposition 2 provides a new view on the driving forces of long-run economic growth. In particular, it establishes that customer base accumulation has the potential to be a quantitatively important contributor to aggregate growth because of its impact on firm-level innovation incentives.

Proposition **3** (Sensitivity to R&D subsidies)
Consider R&D subsidies such that a fraction τ_s of firm-level R&D costs are paid by the government, financed by lump-sum taxes on the household. All else equal
(a) R&D subsidies raise innovation

$$\epsilon_{x,\tau_s} = \frac{\partial x}{\partial \tau_s}\frac{\tau_s}{x} > 0.$$

[15] Note that Proposition 2 focuses on comparing long-run effects, ignoring transitional dynamics.

(b) efficacy of R&D subsidies increases with customer base growth

$$\frac{\partial \epsilon_{x,\tau_s}}{\partial g_b} > 0.$$

(c) efficacy of R&D subsidies increases with the share of R&D expenditures in profits,

$$\frac{\partial \epsilon_{x,\tau_s}}{\partial s_p} > 0, \quad s_p = \frac{Ws}{pc - W(n+s)}.$$

Part (a) of Proposition 3 states that R&D subsidies lead to an increase in innovation rates. This is intuitive as R&D investment becomes cheaper, raising the incentives to innovate.

Next, Part (b) of Proposition 3 shows that the sensitivity of innovation to R&D subsidies is higher when the growth rate of firms' customer bases is strong. The intuition lies in firms' discounting of future profits. With higher growth in the stock of demand, firms place more weight on future periods making them more sensitive to changes in profits brought about by R&D subsidies.

Finally, Part (c) of our last proposition states that R&D subsidies are relatively more effective when innovation expenditures represent a relatively larger share of firm spending. This intuitive result will be important for understanding the quantitative implications of our generalized model in which customer accumulation is endogenous and costly. Overall, Proposition 3 highlights that it is important to take firm-level customer accumulation into account when striving to gauge the efficacy of pro-growth policies.

3 Generalized model

We now relax the assumptions of the previous section – made for analytical convenience – and we generalize the benchmark model in several directions.

3.1 Extensions of the benchmark model

We extend our benchmark framework along three dimensions: (i) endogenous customer acquisition, (ii) generalized scaling of R&D costs and (iii) endogenous entry and exit.

Endogenous customer acquisition: Existing theories. Broadly speaking, there are two alternative theories of customer accumulation. In the first, a firm's customer base increases with past sales (we refer to this as the "pricing model"). In this setting, firms invest into customer accumulation by charging markups below the static optimum which tilts more of the household's consumption towards their good, raises sales and grows the customer base. The alternative theory assumes that a firm's customer base expands with advertising expenditures (we refer to this as the "advertising model").

Both theories have found a wide range of applications in the literature. For instance, pricing models of customer accumulation have been used to understand international prices, firm investment and labor market dynamics (see e.g. Drozd and Nosal, 2012; Gourio and Rudanko, 2014; Kaas and Kimasa, 2021; Rudanko, 2022). Advertising-driven customer acquisition models have been used to study firm dynamics, business cycles or industry life-cycles (see e.g. Dinlersoz and Yorukoglu, 2012; Sedláček and Sterk, 2017; Perla, 2019). Moreover, both theories also find empirical support in different datasets covering various combinations of firm groups, sectors or countries – see e.g. Foster et al. (2016) and Piveteau (2021) for evidence on pricing theories of customer accumulation and e.g. Afrouzi et al. (2021) and Fitzgerald et al. (forthcoming) for evidence in favor of advertising-driven customer acquisition.

As we detail below, we endogenize customer acquisition using the pricing model and quantitatively discipline it with firm-level data. However, in Appendix D.5 we extend our model and let firms also accumulate customers via advertising. Using the model predictions and firm-level data, we show that allowing for advertising does not alter price-driven customer base growth and only strengthens overall endogenous customer accumulation which is central to our results.[16]

Endogenous customer acquisition: Model specification. We follow empirical evidence in Foster et al. (2016) and assume that a firm's customer base is affected by three distinct components. First, the customer base grows "actively" through past sales, $p_j c_j/(PC)$. Second, firms may also increase their customer base via "passive", exogenous, life-cycle profiles, π_j. These serve to capture gradual customer accumulation which occurs without firms' explicit investment, e.g. word-of-mouth effects, customer learning and strengthening of producer-consumer relationships through repeated interactions. Finally, firms are also subject to transitory, exogenous, disruptions to their demand stocks, θ_j. Formally, a firm's customer base and its stock of demand are given by

$$b'_j = (1-\zeta)\left(b_j + \pi_j + \frac{p_j c_j}{PC}\right), \tag{16}$$

$$d_j = \theta_j b_j^{\gamma}. \tag{17}$$

The generalized model, therefore, allows for rich firm-level heterogeneity – stemming from not only life-cycle differences (in customers and productivity) but also transitory disturbances. Such flexibility conforms well with recent evidence which suggests that a

[16] Appendix D.5 also discusses how our model predictions relate to empirical studies of (advertising-driven) customer accumulation. Note, however, that we do not use our model to evaluate welfare. For such questions the distinction between pricing- and advertising-driven customer accumulation is likely to be important, see also e.g. Eslava and Haltiwanger (2021), Edmond et al. (forthcoming).

combination of heterogeneous life-cycle profiles and transitory shocks is indeed needed to account for observed firm dynamics in the data, see Sterk ⓡ al. (2021).

Scaling of R&D costs. For analytical convenience, the benchmark model assumed that R&D intensity is invariant to firm size. However, in the data R&D intensity falls with firm age (see e.g. Akcigit and Kerr (2018) and empirical evidence in the next section). Therefore, we follow Akcigit and Kerr (2018) and specify R&D costs in the generalized model as

$$s_j = \overline{s} n_j^{\sigma} x_j^{\psi}, \tag{18}$$

where $\overline{s} > 0$, $\psi > 1$ and where $\sigma > 0$ controls the degree with which R&D costs scale with size. With $\sigma = 1$, R&D costs scale perfectly with firm size – as in the benchmark model. In the next section, we estimate σ such that our model replicates the empirical relationship between R&D intensity and firm size.

Endogenous firm entry and exit. In order for our generalized model to be able to speak to the process of firm selection and "creative destruction", we endogenize firm entry and exit. In particular, we assume that, at the beginning of each period (i.e. before exogenous transitory shocks, θ_j, realize), firms must pay a per-period fixed cost ϕ (denoted in units of labor) in order to stay in operation. If businesses choose not to pay the cost, they shut down and obtain a return of zero.

At the same time, a mass of potential startups has the option to enter the economy. In order to do so, they must first pay a fixed entry cost κ (denoted in labor units). It is assumed that upon paying the entry cost, startups obtain a random draw of initial values of the idiosyncratic state, z_e, which we assume is identically and independently distributed across firms and time according to a cumulative distribution function $H(z_e)$. After the realization of the initial draws, entrants decide on investment into R&D, x_e and – if they survive – they become incumbent firms in the next period.

3.2 Optimal firm decisions and labor market clearing

Before moving on, let us first define firm-level state variables as $z_j = (q_j, b_j, \theta_j, \pi_j)$ and introduce more compact notation to ease the exposition. In particular, in what follows we will suppress the dependence of firm variables on z_j and instead only use the firm-level subscript j. When our exposition requires it, we make an explicit distinction between successful and unsuccessful R&D outcomes using the subscripts "+" and "−", respectively. For instance, $v'_{j,+} = v(q_j(1+\lambda), b'_j, \theta'_j, \pi'_j)$ denotes next period's firm value of business j, conditional on a successful innovation.

Optimal firm decisions in the generalized framework pertain to pricing, innovation, entry and exit. In addition, we describe labor market clearing, which now also includes labor used for operational and entry costs. All other optimality conditions – household decisions, aggregation and growth – are identical to those of the benchmark model. We defer a formal definition of the balanced growth path equilibrium to Appendix D.

Optimal pricing decisions. Endogenous customer acquisition in the generalized framework introduces a dynamic trade-off into firms' pricing decisions. In particular, businesses can sacrifice current profits – by charging lower markups – in return for the potential to grow their stock of customers and reap greater profits in the future (see Appendix D.5 for an extension allowing for advertising-driven customer accumulation). Formally, the optimal pricing decision is given by

$$\frac{1}{\mu_j} = \frac{1}{\overline{\mu}} + \mathbb{E}\frac{(1-\delta)(1-\zeta)}{1+R'}\left[\frac{1}{\mu_j'} - \frac{1}{\overline{\mu}} + \frac{\gamma}{\eta}\frac{1}{\mu_j'}\frac{m_j'}{b_j'}\right], \tag{19}$$

where $\overline{\mu} = \frac{\eta}{\eta-1}$ is the "static markup", $m_j = \frac{p_j c_j}{PC}$ is the market share of firm j and where the expectation term $\mathbb{E}$ is with respect to the realizations of innovation and customer accumulation shocks.

If $\gamma = 0$, the customer base does not impact a firm's demand and, therefore, businesses have no incentives to cut markups and invest into customer base accumulation. In this case, pricing reverts back to the static optimum, $\overline{\mu}$. In contrast, when $\gamma > 0$, firms have incentives to accumulate customers and they do so by charging markups below the static optimum. Finally, with $\gamma \in (0,1)$ the term $m_j/b_j = \theta_j b_j^{\gamma-1} p_j^{1-\eta}$ decreases as a firm accumulates customers. In other words, a less than full pass-through of customers into demand introduces decreasing returns to customer accumulation.

Put together, the optimal pricing condition (19) implies an increasing life-cycle pattern of markups. New businesses with relatively low levels of customers start with a markup below that of incumbents. This tilts more of the household's consumption towards goods of new firms, allowing them to grow their customer base. As firms age and accumulate customers, they gradually increase markups.[17]

Firm exit. The beginning-of-period firm value (i.e. prior to the realization of transitory shocks) is given by:

$$\tilde{v}_j = \max\left[0, \mathbb{E}_\theta v_j - W\phi\right],$$

[17] A key assumption underlying these predictions regards the demand structure, in particular the constant elasticity of substitution between products in consumption. We adopt the CES structure to be consistent with the vast majority of existing growth models. However, one may consider other demand structures, e.g. that suggested in Kimball (1995) which implies a lower elasticity of substitution for businesses with higher market shares – enabling larger businesses to charge higher markups. In this sense, both demand structures offer the same qualitative implications, albeit for different reasons. That said, Kimball preferences alone would deliver a counterfactually *decreasing* life-cycle profile of markups since in the data it is *young* businesses which are more productive compared more mature firms (see e.g. Foster et al., 2008).

where $\mathbb{E}_\theta$ denotes expectations only over the realizations of stochastic demand shocks, θ_j. The term v_j, therefore, represents firm value conditional on remaining in operation and a particular realization of θ_j:

$$v_j = \max_{p,n,s} \left\{ p_j c_j - W(n_j + s_j) + \frac{1-\delta}{1+R'} \left[x_j \tilde{v}'_{j,+} + (1 - x_j) \tilde{v}'_{j,-} \right] \right\}. \tag{20}$$

Firms decide to shut down when their expected firm value falls below the operational cost. This defines a cutoff value for the idiosyncratic state, z_j^*, and associated firm value, $v_j^* = v(z_j^*)$, where firms are exactly indifferent between shutting down and staying in the economy:

$$\mathbb{E}_\theta v_j^* = W\phi. \tag{21}$$

Note, however, that the idiosyncratic state is a multivariate object making firm selection a complex process depending not only on firm-level productivity, but also on the individual demand stock and its expected evolution. This is an important point to which we return in our quantitative analysis.

Firm entry. Free entry gives rise to the following condition:

$$\kappa W = \int_{z_e} \left\{ -W s_e + \frac{1-\delta}{1+R'} \left[x_e \tilde{v}'_{e,+} + (1 - x_e) \tilde{v}'_{e,-} \right] \right\} dH(z_e). \tag{22}$$

The associated optimal entrant innovation probability, x_e, is then defined by the following condition which mirrors that of incumbent businesses:

$$W\psi \frac{s_e}{x_e} = \frac{1-\delta}{1+R'} \left[\tilde{v}'_{e,+} - \tilde{v}'_{e,-} \right]. \tag{23}$$

Labor market clearing. Finally, let us conclude this subsection by noting that in addition to production employment and workers in R&D, labor is also used for operation and entry costs in the generalized model:

$$N = \int_{j \in \Omega} (n_j + s_j + \phi) dj + \kappa M, \tag{24}$$

where M is the mass of entrants determined by the free entry condition (22).

4 Taking the generalized model to the data

This section describes how we take the generalized model to the data in order to use it for our quantitative analysis in the next section. Before moving on, however, we first describe the different data sources employed in our estimation.

Data sources. Aside from standard aggregate information, we make use of two primary sources of firm-level information when disciplining our model quantitatively: (i) Business Dynamic Statistics (BDS) of the Census Bureau and (ii) Compustat. The key advantage of the BDS is its macroeconomic scope, covering virtually the universe of private-sector employers in the U.S. economy. For this reason, the BDS – with its underlying micro-data, the Longitudinal Business Database – is one of the main data sources for studying U.S. business dynamism (see e.g. Haltiwanger et al., 2013).

However, while the BDS is broad in its coverage, its primary focus is on employment. Therefore, we complement the BDS with information from Compustat which constitutes one of the main source of U.S. firm-level information. While Compustat data has been used extensively in the literature to study the key concepts in our paper – firm-level markups, customer accumulation and R&D (see e.g. Gourio and Rudanko, 2014; De Loecker et al., 2020; Cavenaile et al., 2021) – it is not without its difficulties.

First, it is well known that the Compustat sample of (publicly traded) firms is not representative. To address this issue, when computing model-generated moments with counterparts in Compustat, we use estimated size-based weights which align the Compustat and BDS firm-size distributions. Second, there are certain concerns about the coverage of R&D data in Compustat. For instance, Walmart famously does not report any R&D expenditures in Compustat. Therefore, we explicitly show that the three parameters pinned down using Compustat data ($\bar{s}$, σ and γ) align well with estimates found in other studies using alternative data sources.[18]

4.1 Parametrization: Values set externally

We begin by discussing normalizations and parameters set externally. All model parameters are reported in Table 1.

Common choices. First, consistent with the annual frequency of our firm-level data, we assume the model period to be one year. Therefore, we set the discount factor to $\beta = 0.97$. Second, we set the elasticity of substitution between goods to $\eta = 6$, implying a "static" markup of 20% which falls within the range of values reported in Broda and Weinstein (2006). Third, following the microeconometric evidence on innovation, we set the elasticity of the R&D cost with respect to the innovation probability to $\psi = 2$ (see e.g. Hall et al., 2001; Bloom et al., 2002).

[18] Appendices B and C provide further details on the parametrization, solution method and the estimation of our size-based weights. Note also that when using firm age in Compustat, we consider the time since a particular firm entered the sample. In the Appendix, we show robustness of our results to using age since the founding of a given firm. This definition, however, considerably reduces the sample size.

Normalizations. We set the fixed cost of entry, κ, which controls the mass of firms in the economy, such that aggregate consumption is normalized to $\hat{C} = 1$. Finally, we set the disutility of labor, υ, such that the aggregate wage is normalized to $\hat{W} = 1$.

4.2 Parametrization: Indirect inference

The remaining parameters are estimated jointly using an indirect inference approach in the spirit of Lentz and Mortensen (2008). In particular, we compute various model-generated moments and compare them to their respective empirical counterparts to pin down the remaining parameters by minimizing

$$\min \sum_i \frac{|\text{model}(i) - \text{data}(i)|}{1/2(|\text{model}(i)| + |\text{data}(i)|)},$$

where i indicates a particular moment. While in general all parameters affect model-implied moments, in what follows we discuss our chosen targets in conjunction with the parameters which are most closely linked to them.

Firm-level R&D and aggregate growth. In our model, three parameters are most closely related to firm-level and aggregate productivity growth: the innovation step size, λ, the level of R&D costs, $\overline{s}$, and the degree to which R&D costs scale with firm size, σ. To pin these parameters down, we require our model to match three targets.

First, we make our model match an annual aggregate growth rate of 1.5%, corresponding to U.S. real GDP per capita growth between 1979 and 2019 – consistent with the latest BDS sample. Second, we target the median R&D expenditure to sales ratio of 0.03 in our Compustat sample. This value is consistent with that reported in Akcigit and Kerr (2018), who estimate an R&D intensity of 0.04 based on information from the R&D Survey of the National Science Foundation.

Finally, to discipline the extent to which R&D costs scale with firm size, we follow Akcigit and Kerr (2018) and target a reduced form relationship between firm-level R&D intensity and size:

$$\ln(1+\text{R\&D/sales})_{j,t} = \alpha + \delta_{i,t} + \beta \ln \text{sales}_{j,t} + \epsilon_{j,t}, \tag{25}$$

where $\delta_{i,t}$ are industry-time fixed effects and α is a constant. Using Compustat data, we estimate $\beta = -0.04$, with a standard error of 0.002. Our model replicates this relationship when $\sigma = 1.205$, essentially identical to the estimate in Akcigit and Kerr (2018).[19]

[19] Akcigit and Kerr (2018) estimate that costs of external innovations scale with size (the number of product lines) with $\widetilde{\sigma} = (1-\sigma)/\psi$. The estimates in Table 5 of Akcigit and Kerr (2018) imply that $\widetilde{\sigma} = (1 - 0.395)/0.5 \approx 1.2$.

Table 1: Parameter values

parameter		value	parameter		value
A: Parameters set externally and normalizations					
β	discount factor	0.97	κ	entry cost	0.24
η	elasticity of substitution	6.00	ψ	innovation elasticity	2.00
υ	disutility of labor	1.00			
B: Parameters set via indirect inference					
λ	innovation step size	0.150	ρ_b	$\ln b$, persistence	0.370
$\overline{s}$	R&D efficiency	5.525	σ_b	$\ln b_e$, standard dev.	1.102
σ	R&D cost-size elasticity	1.205	μ_χ	χ, mean	-1.500
γ	demand pass-through	0.630	σ_χ	χ, standard dev.	1.250
ϕ	fixed cost of operation	0.225	ρ_θ	$\ln \theta$, persistence	0.984
δ	exogenous exit rate	0.022	σ_ϵ	ϵ, standard deviation	0.295
σ_q	$\ln q_e$, standard dev.	0.086			

Note: Panel A provides values for parameters set externally to our model as described in the main text. Panel B reports values of parameters set via indirect inference by jointly matching the autocovariance matrix of log employment, the lifecycle profiles of size and exit, the median R&D expenses to sales ratio, aggregate output growth and the R&D intensity-to-size relationship.

Endogenous customer acquisition. Two parameters govern the law of motion for the customer base (16): the customer depreciation rate, ζ, and the pass-through from customer to a firm's demand stock, γ. We set the customer base depreciation rate to 0.4 which lies in between the estimated values of about 0.2 and 0.6 reported in existing studies (see e.g. Gourio and Rudanko, 2014; Foster et al., 2016; Afrouzi et al., 2021).

To parametrize γ, we make use of our model's predictions about pricing dynamics (19). However, instead of attempting to estimate markups in the data which – absent price information – is a notoriously difficult task (see e.g. Bond et al., 2021), we focus on the *observed* sales to costs ratio.[20] To understand this further, we proceed in two steps. First, we show that in order for our model to be consistent with observed sales to costs ratios in the data, markups must increase over firms' life-cycles. Second, we discuss how γ controls such life-cycle patterns of markups.

Towards this end, note that in our model we can write:

$$\frac{p_j c_j}{W(n_j + s_j)} = \frac{p_j}{\frac{W}{q_j}(1 + s_j/n_j)} = \mu_j \frac{1 + \sigma s_j/n_j}{1 + s_j/n_j}, \tag{26}$$

In the data, the sales to costs ratio increases over firms' life-cycles. Specifically, the difference in sales to costs between 20 year old firms and new businesses is about 0.17 log-points.[21] If $\sigma = 1$, (26) implies that also markups would rise with firm age, consistent

[20] Note, however, that subject to the difficulties of estimating markups, there is empirical evidence that estimated markups increase over firms' lifecycles consistent with our model predictions (see e.g. Peters, 2020; Alati, 2021).

[21] Formally, we estimate $y_{j,t} = \sum_a \delta_a + \delta_{i,t}$, where δ_a are age fixed effects, $\delta_{i,t}$ are industry-year fixed effects and $y_{j,t}$ is the variable of interest. We use `xrd/cogs` and `sales/(cogs+xrd)`, respectively, to

with our model predictions. That said, with $\sigma > 1$ (as in our parametrization), the life-cycle pattern of markups may still remain flat if R&D to production costs, s/n, are strongly increasing with firm age. This, however, is not the case empirically. In fact, in the data (and in our model), R&D to production costs are *decreasing* over firms' life-cycles (see Appendix E for details). This is also consistent with Akcigit and Kerr (2018) who find that smaller firms have higher patent to employee ratios. Combining the observed increasing life-cycle pattern of sales to costs with the decreasing pattern of R&D to production costs implies that markups must rise with firm age in order for our model to be consistent with the data.

Finally, recall that with $\gamma = 0$, firms have no incentives to accumulate customers and they would optimally charge constant markups. Therefore, to parameterize γ – which controls the steepness of the life-cycle profile of markups (and therefore the sales to cost ratio) – we target the empirical log-difference in sales to costs of 20 year old firms and new businesses. The resulting value of $\gamma = 0.63$ lies within the range estimated in the data (see Foster et al., 2016). Moreover, in Subsection 4.3, we further show that the customer base dynamics implied by our model are in line with existing empirical estimates, suggesting the realism of the model's driving forces. In addition, Appendix D.5 shows that allowing firms to also accumulate customers using advertising does not change the resulting life-cycle profile of markups.

Firm exit. In our model, businesses shut down endogenously if they cannot afford to pay the fixed operational cost, ϕ. In addition, each business faces an exogenous rate of exit, δ. We discipline these two parameters by making our model match the life-cycle profile of firm exit in the BDS, depicted in Figure 1 in Panel (a). While δ mainly influences the overall level of firm exit, the operational cost – in combination with firm-level heterogeneity described below – helps match the declining pattern with firm age. The latter occurs as surviving businesses gradually grow and increase their firm value, lowering the risk of not affording the operational cost.

Transitory shocks and passive life-cycle profiles. Finally, to pin down the nature of exogenous firm-level heterogeneity, θ_j and π_j, we follow Sterk ⓡ al. (2021). The latter document that, in addition to the life-cycle profile of firm size, important information about the sources of firm heterogeneity is contained in the autocovariance structure of firm-level employment. Therefore, we make our model match these two sets of empirical moments. Towards this end, we assume that the exogenous transitory shocks follow an AR(1) process in logs:

$$\ln \theta'_j = \rho_\theta \ln \theta_j + \eta_j, \tag{27}$$

measure s/n and $pc/(n+s)$ in Compustat. Appendix E provides further details, including the full estimated life-cycle profiles.

where all startups are assumed to enter with $\theta_e = 1$, ρ_θ is a persistence parameter and where η_j are idiosyncratic shocks, distributed identically and independently over time and across firms with zero mean and variance σ_θ^2.

Next, instead of explicitly parametrizing the exogenous component of customer base accumulation, π_j, it is computationally more convenient to proceed in two steps. First, we use firms' optimal pricing decisions (19) to determine sales and, in turn, the evolution of the *endogenous* component of a firm's customer base:

$$b'_{j,\text{endo}} = (1-\zeta)\left(b_{j,\text{endo}} + \frac{p_j c_j}{PC}\right).$$

Second, we specify a law of motion for the customer base *as a whole*, b_j, and back out the life-cycle profile of its exogenous component as a non-parametric wedge from $b_{j,\text{exo}} = b_j - b_{j,\text{endo}}$, where $b'_{j,\text{exo}} = (1-\zeta)(b_{j,\text{exo}} + \pi_j)$.[22] Following the specification in Sterk ⓡ al. (2021), we assume that a firm's overall customer base evolves according to:

$$\ln b'_j = \chi_j + \rho_b \ln b_j, \tag{28}$$

where χ_j is a firm-specific constant and where ρ_b is a persistence parameter. We assume that χ_j and the initial value of b_j – which are both drawn by startups at the time of entry – are independently and identically distributed over time and across firms with respective means μ_χ and μ_b and variances σ_χ^2 and σ_b^2.

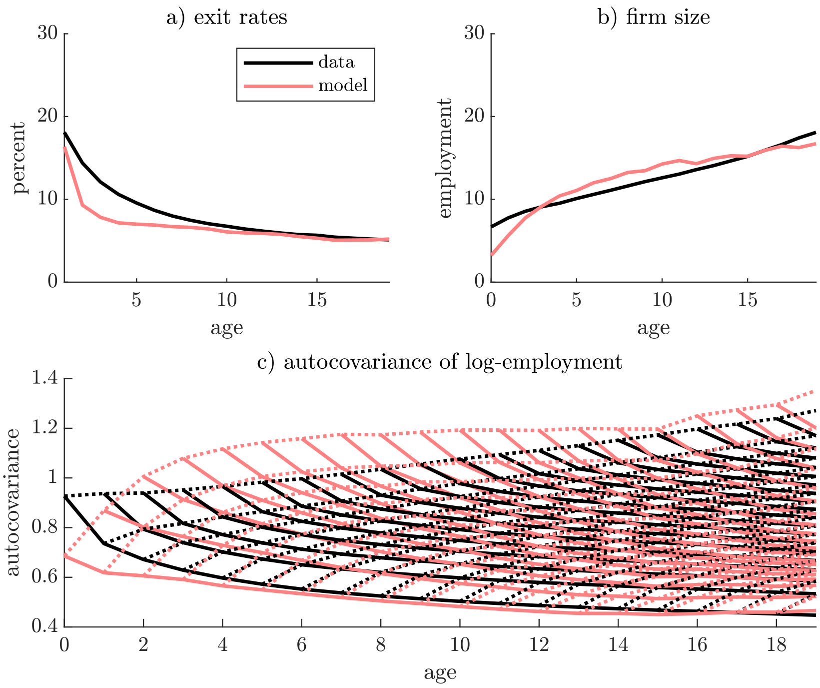
4.3 Model performance

Before turning to our quantitative results, we document how our model matches the targets and a range of untargeted moments.

Targeted moments. Figure 1 displays the model's performance along three sets of targeted moments: life-cycle profiles of firm (i) exit rates, (ii) size, and (iii) the auto-covariance structure of log-employment. In addition, Panel A of Table (2) documents how the model matches aggregate growth, the median R&D-sales ratio, the firm-level R&D-to-size relationship and the sales to costs ratio between mature firms and startups.

[22]Note that $b'_{j,\text{endo}} + b'_{j,\text{exo}} = (1-\zeta)(b_j + \pi_j + p_j c_j/(PC)) = b'_j$ as in (16). The computational convenience stems from the fact that – for the purposes of parametrizing the model – this approach enables us to ignore the feedback between customer accumulation and productivity. This eases the parametrization process considerably, because the endogenous feedback loop between q and b can make the model sensitive to certain parameter combinations. Appendix D shows the time-paths of the endogenous and exogenous customer bases, highlighting that it is the endogenous component which is responsible for the vast majority of customer acquisition. In fact, after firms' second year, the exogenous component tends to *reduce* firms' customer base on average. One interpretation of such patterns is that π_j captures the impact of other frictions (e.g. financial or informational) which hold back firms' growth.

Figure 1: Model fit



Note: Top panels show average exit rates and firm size (employment) in the model and BDS data. The bottom panel shows the observed and model-implied autocovariance matrix of log employment for a balanced panel of firms surviving at least up to age 19.

Untargetted moments. Figure 2 shows that the generalized model is also consistent with the employment distribution by firm size and age. Panel B of Table 2 documents that our model also replicates the empirical patterns of job creation and destruction, including those pertaining to firm entry and exit.

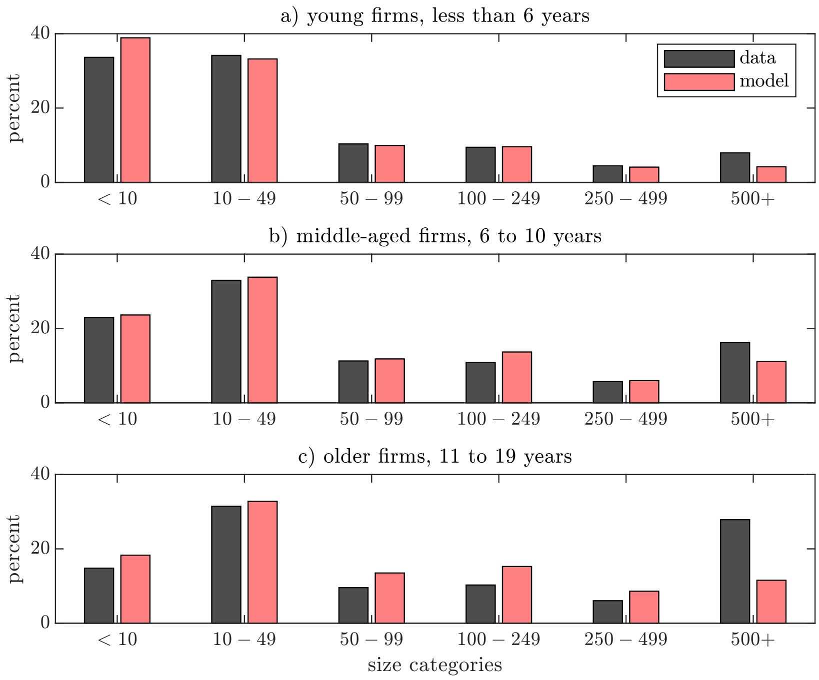
In addition, we further gauge the realism of our model-implied customer base dynamics – the key novel feature of our framework. First, and as noted in the paragraphs above, our estimated value of "pass-through" from firms' customer bases to demand, γ, falls within the range estimated in the data. Second, we draw upon the methodology and empirical evidence presented in Foster et al. (2008). Specifically, the authors estimate firm-level "demand shocks", $\widetilde{d}_{j,t}$, as residuals from the following regression[23]

$$\ln c_{j,t} = \beta_0 + \beta_1 \ln p_{j,t} + \widetilde{d}_{j,t}. \tag{29}$$

Notice that $\widetilde{d}_{j,t}$ – as estimated by Foster et al. (2008) – exactly represents the logarithm of firms' demand stocks, $\ln d_{j,t}$, in our model. Therefore, estimating (29) on simulated data provides a direct comparison of the model-implied customer base dynamics to their empirical counterparts. Encouragingly, the model estimates of the persistence and standard deviation of $\widetilde{d}_{j,t}$ are very close to those in the data (see bottom of Table 2).

[23]The authors use physical output and instrument firm-level prices with estimates of physical productivity, TFPQ. In our model, we observe prices directly and, therefore, do not need a two-step procedure. Nevertheless, following their procedure completely, changes little on our results.

Figure 2: Firm size-age distribution: data and model



Note: Size-age distribution in the BDS data and in the model. Both distributions are expressed as shares of employment in a given age category (young: < 6 years, middle-aged: $6 - 10$ years and older: $11 - 19$ years).

Table 2: Model fit

	model	data
A: Targeted moments		
aggregate growth rate	1.5%	1.5%
median R&D-output ratio	2.3%	3.0%
firm-level R&D-size relationship	-0.03	-0.04
sales to costs ratio, mature firms vs startups	0.18	0.17
B: Untargeted firm dynamics moments		
job creation rate	22%	17%
job destruction rate	22%	15%
job creation share from entry	9%	9%
job destruction share from exit	17%	17%
firm-level demand stock, persistence	0.91	0.91
firm-level demand stock, standard deviation	1.19	1.16

Note: Panel A shows three targeted moments: Aggregate (real GDP) growth, R&D expenditure to sales ratio, the firm-level R&D intensity to size relationship, estimated in (25) and the difference in sales to costs ratio between 20 year old firms and startups. The latter three targets are all estimated from Compustat. Panel B shows untargeted moments pertaining to firm dynamics (using data from the BDS). The last two rows show the persistence and standard deviation of "demand shocks" as estimated by Foster et al. (2008), see Tables 1 and 3, respectively.

5 Quantitative analysis

In this section, we use our parameterized model for two purposes. First, we quantify how customer acquisition affects firm-level incentives to conduct R&D and, in turn, aggregate growth. Second, we demonstrate that models which ignore customer accumulation provide significantly different conclusions about the efficacy of growth policies.

5.1 Customer acquisition and economic growth

Section 2.2 showed that the presence of customer acquisition increases firms' incentives to conduct R&D and, in turn, partly drives aggregate growth. In what follows, we analyze these effects in the generalized model, quantifying Propositions 1 and 2. In doing so, we highlight a new feedback loop between *endogenous* customer accumulation and productivity growth which turns out to be quantitatively powerful. First, however, we describe how we quantify the impact of customer acquisition on model results.

Quantifying the impact of customer accumulation. To isolate the impact of customer acquisition on firm-level and aggregate outcomes, we focus on business-level R&D decisions. Specifically, we take our baseline economy and solve for *counterfactual* innovation decisions, $\underline{x}_j$. The latter are made under the same equilibrium conditions and firm-level state variables found in our generalized model, but with firms – counterfactually – assuming that their respective customer base levels are fixed, i.e. $b'_j = b_j$.

Note that counterfactual R&D decisions feed into other model outcomes. This happens because R&D decisions impact not only firms' productivity levels, but through their effect on sales also firms' customer accumulation, see (17). Therefore, using the counterfactual R&D decisions, we can compute counterfactual values for both firm-level state variables:

$$\underline{q}'_j = \begin{cases} q_j(1+\lambda) & \text{with probability } \underline{x}_j, \\ q_j & \text{with probability } 1-\underline{x}_j, \end{cases}$$
$$\underline{b}'_j = (1-\zeta)\left(b_j + \pi_j + \frac{p(\underline{q}_j)c(\underline{q}_j)}{PC}\right),$$

where firms' counterfactual demand stocks are given by $\underline{d}_j = \theta_j \underline{b}_j^{\gamma}$.[24]

[24] An alternative would be to assume that firms understand the underlying structure of demand, but *expect* their future customer base to be unchanged, i.e. $\mathbb{E}[b'_j] = b_j$. We do not prefer this specification as it implicitly assumes that transitory shocks adjust in a particular way along firms' life-cycles to ensure that expected demand stock equals current values. Note further that when computing counterfactual outcomes, we only focus on the impact of R&D decisions and leave firm exit and pricing decisions equal to those from the baseline. Finally, while our primary interest lies in the impact of customer accumulation, Appendix D quantifies the impact of transitory shocks, θ, on firm-level outcomes.

Table 3: Decomposition of firm-level innovation rates

	Baseline	Contribution of customer acquisition	
		Percentage points	Share of baseline
	x	$x - \underline{x}$	$(x - \underline{x})/x$
All firms	6.92	1.31	0.19
Young firms	7.51	3.13	0.42

Note: The first column shows "baseline" average innovation rates, x, for all firms (top row) and young business not older than 5 years (bottom row). The second and third columns report the contributions of customer acquisition to innovation rates in "percentage points", i.e. $x - \underline{x}$, and as a "share of baseline" values, i.e. $(x - \underline{x})/x$. All values are in percent.

Finally, using $\underline{q}$ and $\underline{d}$, we can compute all other counterfactual model outcomes. The contribution of customer base acquisition to a particular endogenous variable is then given by the difference between its baseline and counterfactual values. For example, $(x-\underline{x})/x$ is the fraction of firm-level innovation rates driven by the presence of customer acquisition.

Customer acquisition and firm-level R&D. Proposition 1 explains that the presence of customer accumulation increases the returns from innovation. This happens because customer accumulation comes with expansions of firms' market shares, generating a *firm-level market size effect.*

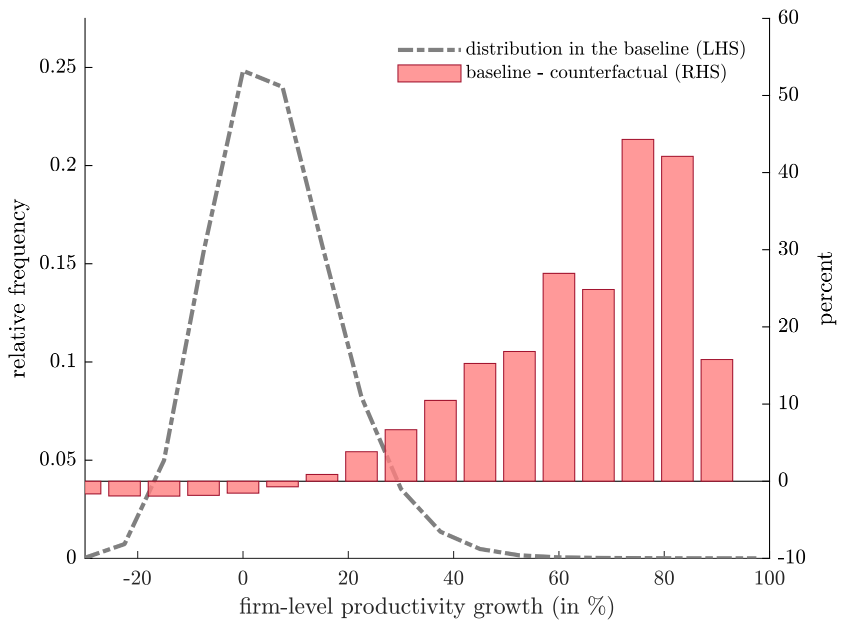
This channel is present also in the generalized model. For instance, the correlation between firms' innovation rates and their demand stock *growth* – which partly drives firms' market size expansions – is 0.71. This tight relationship is further highlighted in Table 3. The first column shows average innovation rates among all and young businesses (not older than five years). The latter exhibit higher innovation rates on average, primarily owing to stronger customer base growth.

The second and third columns document that the prospect of customer acquisition raises firm-level innovation rates, as predicted by Proposition 1. Customer acquisition accounts for slightly more than 1/5 of average innovation rates among all firms. This effect is much stronger among young businesses where customer acquisition accounts for over 40% of average innovation rates.

Feedback loop between productivity and customer acquisition. Let us now describe a key novel channel in our model – an endogenous feedback loop between productivity and customer acquisition. Recall that firms with higher productivity can afford to charge lower prices and, therefore, increase sales. This, in turn, leads to faster customer accumulation. At the same time, however, expected customer accumulation (and the associated growth in firms' market size) provides incentives to conduct more R&D (see Proposition 1). Therefore, endogenous customer accumulation reinforces R&D and vice versa.

This endogenous feedback loop between customer acquisition and productivity has

Figure 3: Feedback loop between productivity growth and customer acquisition



Note: The bars show the percentage difference in customer base distribution between the baseline and that implied by firms' counterfactual R&D decisions, i.e. $(b-\underline{b})/b \times 100$. The dashed-dotted line displays the baseline customer base distribution. Both are expressed as a function of firm-level productivity growth, $(q'/q-1) \times 100$.

important aggregate consequences going beyond firm-level decisions. This is because faster customer accumulation does not only reinforce incentives to conduct R&D, but it also shifts market shares towards businesses experiencing high productivity growth – often referred to as "gazelles."

Figure 3 visualizes this reallocation effect. Specifically, the bars show the difference in the customer base distribution in the baseline relative to that implied by firms' counterfactual innovation decisions, $(b-\underline{b})/b$, as a function of firm-level productivity growth. In addition, to understand where the mass of firms is located, the dashed-dotted line displays the baseline customer base distribution itself.

As can be seen from the figure, the presence of customer acquisition increases market shares for businesses experiencing fast productivity growth. Our framework, therefore, suggests that the documented importance of gazelles for aggregate outcomes (see e.g. Haltiwanger et al., 2016) may partly be the result of their customer accumulation, rather than exclusively being due to their superior productivity growth.

Finally, note that these changes in firms' market shares are related to, but distinct from, the reallocation effects described in Lentz and Mortensen (2005, 2008). The latter studies stress that reallocation of workers from less to more productive firms is an important source of aggregate growth. In our framework customer accumulation amplifies the reallocation of resources towards businesses with higher productivity *growth*. More importantly, however, because market shares are partly driven by customer accumula-

Table 4: Decomposition of aggregate growth

	Baseline	Contribution of customer acquisition		
		Innovation	Customer base	Overall
	g	$g - g(d, \underline{q})$	$g - g(\underline{d}, q)$	$g - g(\underline{d}, \underline{q})$
Percentage points	1.46	0.12	0.42	0.55
Share of baseline	1.00	0.08	0.29	0.38

Note: The first column reports aggregate growth in the "baseline" model. The second, third and fourth columns show the contribution of customer acquisition to aggregate growth divided into three effects: (i) "innovation", (ii) "customer base" and (iii) "overall". The innovation effect computes growth with baseline demand stocks, but counterfactual productivity, $\underline{x}$, the customer base effect computes growth with counterfactual demand stocks, $\underline{d}$, but baseline innovation rates and the overall effect computes growth using both counterfactual demand stocks and innovation rates. The top row reports growth (contributions) in percentage points, while the bottom row reports values relative to baseline growth.

tion, reallocation of market shares occur even in the absence of differences in firm-level productivity.

Customer acquisition and aggregate growth. We now turn to quantifying the impact customer acquisition has on aggregate economic growth. Towards this end, we compute counterfactual aggregate growth rates as follows:

$$g(\underline{d}, \underline{q}) = Q(\underline{d}', \underline{q}')/Q - 1,$$

where we have made explicit the dependence of the aggregate productivity index (15) on firm-level demand stocks and productivity levels. In computing counterfactual growth rates, we can further distinguish between three cases which we refer to as (i) innovation, (ii) customer base and (iii) overall effects of customer acquisition on aggregate growth. Intuitively, the innovation effect considers only the channel operating through productivity, $g(d, \underline{q})$, the reallocation effect considers only the channel operating through demand stocks, $g(\underline{d}, q)$, and the overall effect considers both channels.

Table 4 reports the results of our decomposition. While the first column shows the baseline growth rate, the second, third and fourth columns report the different contributions of customer acquisition to aggregate growth. We do so separately for the three cases discussed above. The top row reports growth contributions in percentage points, $g - \underline{g}$, while the bottom row reports contributions as a share of baseline growth, $(g - \underline{g})/g$.

The results suggest that the presence of customer acquisition is a quantitatively strong force driving aggregate growth. In particular, the overall effect shows that more than 1/3 of aggregate economic growth is accounted for by customer acquisition. The presence of customer accumulation raises firms' incentives to conduct R&D. This increases the speed of productivity growth and directly translates into aggregate growth. However, the second column of Table 4 shows that this direct innovation effect accounts for "only" about 1/5 of the overall impact of customer acquisition on growth.

Instead, it is the indirect, customer base, effect which is responsible for the majority of the overall impact. Because higher R&D incentives affect mainly businesses with high productivity growth (see Figure 3), these businesses subsequently experience the strongest customer base growth. This shifts market shares (and hence resources) towards gazelles further amplifying the overall impact on aggregate growth. Therefore, while aggregate growth has typically been considered a purely supply-side phenomenon, our framework puts forward a novel – and quantitatively important – driver of aggregate growth in the form of customer acquisition.

5.2 Implications for growth policies

The previous subsection quantified that firm-level customer acquisition accounts for more than 1/3 of aggregate economic growth. In this subsection, we stress that customer acquisition fundamentally changes not only the level of growth, but also its responsiveness to policy interventions.

The policies we consider in this subsection should not be viewed as instruments aimed to bring the economy closer to a first best allocation.[25] Instead, we use them to quantify that a model which is observationally equivalent to our generalized framework, but which ignores customer accumulation, provides very different responses to policy interventions. As will become clear below, this happens because firm selection and investment patterns – which determine aggregate growth and which help shape the economy's response to policies – are very different in our generalized model compared to one which ignores customer acquisition.

A "restricted" economy. In order to quantify how the presence of firm-level customer acquisition changes the responsiveness of growth to policies, we compare our generalized model to an identical framework which, however, ignores customer acquisition, i.e. $d_j = d$ for all j. Formally, we restrict our generalized model such that $\sigma_\epsilon = \sigma_\chi = \sigma_b = 0$, but recalibrate the economy to match the same targets as described in Section 4.[26]

In what follows, we refer to the latter model as the "restricted" economy. This approach allows us to show that an economy which is – in many dimensions – observationally equivalent to our generalized model, but which ignores customer accumulation, *does not* in fact provide the same quantitative predictions regarding the macroeconomic impact of policies.

[25] See Appendix A for a derivation of the planner's R&D allocations.

[26] Specifically, the restricted model matches the same targets as the generalized framework, with the exception of the autocovariance structure of firm-level employment which – in the absence of richer heterogeneity – is hard to match. Not recalibrating the restricted economy would dramatically alters firms' life-cycle profiles and, in turn, aggregate outcomes. Such an exercise, therefore, does not offer a meaningful comparison to the baseline economy. Appendix D provides more details on the restricted economy, its parameterization and solution as well as additional results.

Introducing subsidies to R&D and operational costs. We introduce two policy instruments into both the baseline and restricted economies: (i) R&D subsidies, τ_s and (ii) subsidies to costs of operation, τ_ϕ. We assume that both policies are financed with lump-sum taxes on households and provided to all businesses at the same rate. Formally, firm values become:

$$v_j = \max_{p,n,s} \left\{ p_j c_j - W(n_j + s_j(1-\tau_s)) + \frac{1-\delta}{1+R'} \left[x_j \tilde{v}'_{j,+} + (1-x_j)\tilde{v}'_{j,-} \right] \right\},$$

where $\tilde{v}_j = \max \left[0, \mathbb{E}_\theta v_j - W\phi(1-\tau_\phi) \right]$. In our analysis, we consider two cases. One with positive R&D subsidies (but zero operational cost subsidies) and one with positive subsidies to costs of operation (but zero R&D subsidies). In doing so, we ensure that in both the baseline and restricted economies subsidies account for the same fraction of output.

Finally, we restrict our analysis to the long-run, abstracting from transitional dynamics. We do, however, consider general equilibrium effects. Therefore, following the introduction of the respective policies, we recompute equilibrium firm entry, wages and aggregate growth for each of the economies.

Table 5 reports the results in the form of long-run changes induced by the respective policies. Given our focus on growth, we report three key variables: (i) the aggregate growth rate (first column), (ii) average firm exit rates (second column) and (iii) average firm-level innovation rates (3rd column).

Operational cost subsidies and firm selection. Let us first consider the impact of subsidizing firms' operations, $\tau_\phi > 0$, in the restricted economy. Lower operational costs make it easier for businesses to survive. However, it is firms with relatively low productivity which are now able to survive under the policy. As a result, innovation and aggregate growth decline.

The effects are qualitatively the same in the generalized model, where aggregate growth also declines in response to subsidies to firms' operations. However, the response of the generalized model is only about half as strong as that in the restricted economy. In particular, while aggregate economic growth slows by 0.07 percentage points in the restricted model, it declines by only 0.04 percentage points in the baseline.

The reason for this stark quantitative difference lies in the process of firm selection, which is very different in the two economies. Figure 4 shows the distribution of incumbent firms in the restricted (Panel A) and baseline (Panel B) models, respectively. In both models, firms decide whether or not to stay in the economy based on their firm values, see (21). In the restricted economy, however, firm values are determined solely by firm-specific productivity. Therefore, firm selection necessarily happens only on this dimension

Table 5: Impact of policies on aggregate growth

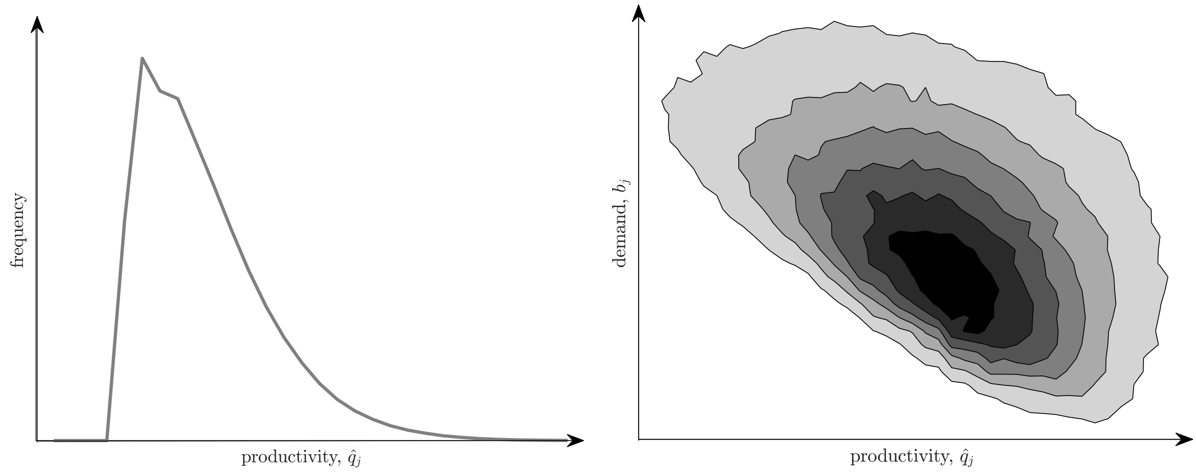
	Restricted model	Generalized model
Subsidies to operational costs, $\tau_\phi > 0$	-0.07	-0.04
Subsidies to R&D expenditures, $\tau_s > 0$	$+0.04$	$+0.11$

Note: The table shows general equilibrium effects of subsidies to firms' operational costs (top row) and to R&D costs (bottom row). The table reports results for the generalized model and the restricted economy which abstracts from customer accumulation. The table depicts effects on aggregate growth in percentage point deviations from the respective values without policies.

Figure 4: Distribution of incumbent firms

(A) Restricted model

(B) Baseline model



Note: The figure shows the distribution of incumbent firms in the restricted (Panel A) and baseline (Panel B) models. In the baseline model, firms select on a combination of demand stocks and productivity with darker shades indicating more densely populated areas. In the restricted model, firms select purely on productivity.

with businesses below a particular productivity threshold deciding to shut down.

In contrast, firm values in the baseline economy are determined by a combination of productivity levels and demand stocks. Panel B of Figure 4 shows a clear negative relationship between the two among surviving firms. Intuitively, if a firm enjoys a high demand stock, it can survive despite having relatively low productivity and vice versa.[27] Therefore, the overall effect of subsidizing operational costs in the baseline model depends on how exactly it shifts the *joint* distribution of firm-level demand stocks and productivity levels among surviving businesses.

For our parametrization of the baseline model, average firm-level innovation rates increase slightly. This offsets some of the negative effects of less stringent selection (lower exit rates) and the resulting quantitative impact on aggregate growth is small.

[27]In the Appendix D.3, we show that firm exit is in fact primarily determined by demand-side factors in the baseline model, consistent with empirical evidence provided in Foster et al. (2008).

R&D subsidies and endogenous customer acquisition. As a second policy we consider subsidies to R&D costs, τ_s. Let us again begin by focusing on the effects in the restricted model. R&D subsidies make investing into productivity cheaper and, therefore, they raise firm-level innovation rates. This, in turn, directly translates into faster economic growth. Once again, while the results are qualitatively the same in the generalized model, they differ quantitatively. This time, however, it is the generalized model which is about twice as more sensitive to the policy compared to the restricted economy.

Recall from Proposition 3 that there are two counter-acting forces at play in the generalized model. On the one hand, customer accumulation and the associated firm-level market size effect increase the sensitivity of firms to R&D policies. On the other hand, since customer accumulation is costly, firms with lower R&D-to-profit ratios will respond less to R&D subsidies.

Patterns in the generalized model confirm both of these theoretical predictions. For instance, young businesses – which on average enjoy stronger customer base growth – respond to the R&D subsidy about 50 percent more strongly compared to mature firms. In contrast, businesses with below-median R&D expenditures-to-profit shares respond about 1.5 less to the subsidies compared to above-median firms. Under our parametrization, the first channel dominates. Therefore, the presence of customer acquisition makes firms more responsive to R&D subsidies, raising aggregate growth more than twice as much compared to an economy which ignores customer accumulation.

The two examples discussed in this subsection highlight that ignoring demand-side factors – as is common in existing endogenous growth models – can not only skew our understanding of the drivers of growth, but also distort our view of the effectiveness of pro-growth policies. While we have focused on two specific policies, the underlying reasons for the associated quantitative results are more general. Specifically, the presence of firm-level customer acquisition changes the nature of firm selection and the distribution of investment costs. Therefore, any policies or shocks which affect business dynamism will propagate differently through an economy which ignores customer acquisition compared to our baseline framework.

6 Empirical support for model predictions

As a final step in our analysis, we provide empirical support for some of our key model predictions. First, we document evidence for a link between customer accumulation and R&D decisions, a central feature of our model. Second, we document that responses of firm-level R&D to subsidies in the data follow the same heterogeneous patterns as predicted by our model (Proposition 3 and Section 5.2).

Our primary source of firm-level information is Compustat with a sample running from 1950 to 2019. Following common practice and consistent with the parameterization of the generalized model, we exclude foreign firms, utilities and financial companies. Appendix E contains further details on sample selection, estimation as well as a range of additional empirical results and robustness exercises.

6.1 Customer accumulation and firm-level R&D

A central theme in our model is a link between customer accumulation and R&D decisions. In particular, firms facing greater incentives to accumulate customers also invest more into R&D. We now test this relationship in the data.

Estimation. In our model, incentives to accumulate customers are governed by the strength of pass-through from customers to demand, γ. Recall that with $\gamma = 0$, firms have no reason to accumulate customers, while $\gamma > 0$ provides incentives for investing into customer base growth. Proposition 1 then explains that businesses facing stronger expected customer base growth invest more into R&D.

While the generalized model does not explicitly consider heterogeneity in γ, Proposition 1 predicts that, all else equal, firms operating in markets with stronger customer accumulation motives (higher γ) would invest more in R&D. Interpreting a market as a 3-digit NAICS industry, we test this prediction by estimating the following regression:

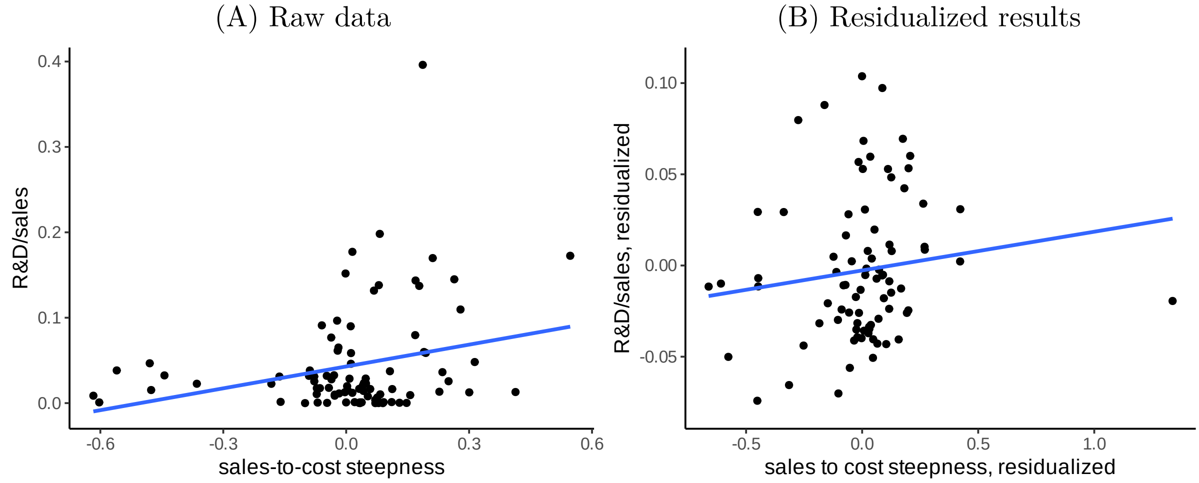
$$\log(1 + \text{R\&D/sales})_i = \delta + \alpha M_i + \Lambda X_i + \epsilon_i, \tag{30}$$

where i indicates a particular industry, M_i is our measure of customer acquisition incentives and X_i are additional controls described below. To measure customer acquisition incentives, we again rely on the life-cycle steepness of sales to costs – the same procedure used to parameterize our model.[28] The key coefficient of interest is α, which measures the (conditional) cross-sectional correlation between R&D expenditures and the strength of customer acquisition incentives.

Results. The left panel of Figure 5 shows the "raw" relationship between average life-cycle sales-to-cost steepness and average R&D intensity at the 3-digit industry level. There is a clear positive (and statistically significant) relationship between the two, as predicted by our model.

[28] Average life-cycle steepness in sales to costs are defined as the average log-difference in sales to costs of firms aged 20 and 0 years. Appendix E lists industries with high and low steepness of sales to costs, pointing to a relatively intuitive ranking. For instance, while Repair and Maintenance, Motor Vehicle and Parts Dealers or Special Trade Contractors display high steepness of sales to costs (i.e. reflecting the need of businesses in these industries to invest into customer accumulation), Oil and Gas Extraction, Textile Mills or Fabricated Metal Product Manufacturing have low sales-to-cost steepness. The Appendix also shows that our results are robust to first estimating firm-level life-cycle profiles of sales to costs and then using their average. We also obtain similar results when measuring "true" age using firms' founding year, rather then the year of their IPO. In addition, we also show that similar results are obtained when considering selling expenditures as a measure of investment into customer accumulation.

Figure 5: Industry variation in R&D intensity and life-cycle steepness of sales to costs



Note: The figure shows the relationship between average R&D intensity (log(1+`xrd`/`sales`)) and life-cycle steepness of sales to costs across 3-digit SIC industries. Panel A does so using "raw data". Panel B, "residualized results", first regresses both variables on average leverage (debt to equity), the Herfindahl index and average firm size (employment) at the industry level and plots the respective residuals. The estimated regression coefficients (standard errors) are 0.09 (0.03) and 0.05 (0.02) in Panel A and Panel B, respectively.

While the "raw" relationship discussed above aligns well with our theory, there may be other factors influencing it. For instance, market-level differences in the severity of financial frictions, extent of market concentration or fixed costs may partly play a role. To alleviate concerns that the documented cross-section support for our theory stems from an empirical correlation between sales to costs steepness and (proxies of) financial frictions, market power or fixed costs we now explicitly control for the latter three factors. In particular, we regress R&D intensity across markets on life-cycle steepness of sales to costs, but also control variables X_i which include average leverage (debt-to-equity), the Herfindahl index of industry concentration and average firm size (employment).

The right panel of Figure 5 displays the resulting "residualized" relationship between life-cycle steepness of sales to costs and R&D intensity. While the three factors we consider partly explain R&D intensity (see Appendix E for details), they do not change its positive (and statistically significant) relationship to the steepness of sales to costs.[29] Therefore, the provided empirical evidence not only supports our proposed mechanism, but also suggests that it is more-or-less orthogonal to other potential alternatives.

[29]The estimated regression coefficients (and standard errors) between R&D intensity and the steepness of sales to costs across industries are, respectively, 0.09 (0.03) in the raw data and 0.05 (0.02) when controlling for other factors.

6.2 Firm-level impact of R&D subsidies

Finally, we turn our attention to model predictions regarding the heterogeneous firm-level impact of R&D subsidies.

Estimation. Our model predicts that R&D subsidies raise firm-level innovation expenditures. Moreover, this effect is stronger for young firms, which on average enjoy faster customer acquisition, but milder for businesses with lower R&D-to-profit ratios – see Proposition 3 and Section 5.2. To test these predictions, we estimate the following regression:

$$\begin{aligned}\log(\text{R\&D})_{j,t} =& \alpha\tau_{s,t} + \beta_a \mathbb{1}_{\text{age}_{j,t}\leq 5} \times \tau_{s,t} + \beta_b \mathbb{1}_{\bar{s}_{p,j}>\bar{s}_p^{\text{median}}} \times \tau_{s,t} \\ &+ \Gamma X_{j,t} + \delta_j + \delta_s + \delta_{i,t} + \epsilon_{j,t},\end{aligned} \tag{31}$$

where δ_j, δ_s and $\delta_{i,t}$ are, respectively firm, state and sector-time fixed effects, $\tau_{s,t}$ is our measure of R&D costs, $\mathbb{1}_{\text{age}_{j,t}\leq 5}$ denotes a dummy variable equal to one if the firm is "young" (less than 6 years of age). Furthermore, guided by our theoretical results in Part (c) of Proposition 3, we separate firms into groups of above- and below-median in terms of their average R&D expenditures-to-profits ratio, $s_p = \frac{Ws}{pc-W(n+s)}$. Therefore, $\mathbb{1}_{\bar{s}_{p,j}>\bar{s}_p^{\text{med}}}$ denotes a dummy variable equal to one if a firm – on average in our sample – is characterized by an above-median R&D expenditures-to-profits ratio.[30] Our model predicts that firms with below median R&D expenditures-to-profits respond less strongly to R&D subsidies because they simultaneously invest relatively more into customer accumulation.

Data and results. In the U.S., R&D incentives typically take the form of tax credits, corresponding to a certain fraction of tax-deductible R&D expenditures. As explained in Wilson (2009), U.S. states differ in the extent and inception dates of R&D tax incentives. Therefore, we use this variation in the relative cost of innovation over time and across U.S. states to estimate (31). However, since R&D tax credits are themselves taxable income – and the latter is subject to different local and federal taxes – we use the "user cost of R&D" variable reported in Wilson (2009). This measure uses the statutory tax credit rate, but adjusts it for local and federal corporate tax rates, recapture and R&D capital depreciation.

Table 6 shows the results, noting that a higher user cost of R&D corresponds to *lower* R&D subsidies. As predicted by our model, R&D subsidies raise firm-level ex-

[30] In Compustat, we measure s_p as `xrd/(sale-xrd-cogs)`, where `xrd` are R&D expenses, `sale` marks revenues, and `cogs` stands for the cost of goods sold. Our results are robust to using operating income as a measure of profits in the denominator. In estimating firm-specific averages of the latter ratio, we control for industry-year fixed effects. We use the median split on average rather than contemporary R&D expenditures-to-profits ratio to avoid a mechanical relationship between this share on the right-hand side of the estimated equation and the R&D expenses on the left-hand side.

Table 6: R&D tax credit and firm-level R&D expenses

	R&D expenses		
	(i)	(ii)	(iii)
R&D user cost	-1.94**	-1.66*	-0.683
	(0.949)	(0.897)	(0.694)
young firm	-0.075**	0.913***	0.781***
	(0.034)	(0.232)	(0.197)
R&D user cost × young firm		-0.740***	-0.633***
		(0.170)	(0.143)
R&D user cost × high R&D share			-1.19***
			(0.178)
additional controls	✓	✓	✓
firm fixed effects	✓	✓	✓
time × industry fixed effects	✓	✓	✓
state fixed effects	✓	✓	✓
observations	60,371	60,371	57,483

Note: The table reports coefficient estimates from (31). Additional controls include log revenues and financial variables: liquidity ratio (debt to current assets), leverage (debt to equity), and the log of dividend paid. Moreover, we control for overall state level corporate income tax, employment growth, and output growth at the state level. Stars indicate p-value, "$*$"= 0.1, "$**$"= 0.05, "$***$"= 0.01. Standard errors clustered at the state level in parentheses.

penditures on innovation (first row, first column). Moreover, all else equal, they do so more for younger businesses (third row, second column) which, on average, experience faster demand stock growth. In contrast, all else equal, firms with below median R&D expenditures-to-profits respond less strongly to R&D subsidies (fourth row, third column).

The empirical results presented above are consistent with our model predictions that customer base accumulation is closely linked to firms' R&D investment decisions. Therefore, the former should not be ignored when studying innovation at the firm and aggregate levels.

7 Concluding remarks

Firm life-cycle dynamics are a central force in endogenous growth models. In existing growth models, such dynamics are typically determined purely by productivity differences between firms. Instead, recent and expanding evidence suggests that demand-side factors, in particular active customer base growth, are crucial for driving firm-level outcomes. In this paper, we build on this evidence and propose a novel endogenous growth model featuring customer acquisition. The presence of customer acquisition increases firms' incentives to conduct R&D and endogenously reallocates market shares and resources towards businesses experiencing fast productivity growth. Combined, these effects serve as a quantitatively important driver of aggregate growth. Moreover, ignoring customer acquisition changes the quantitative response of the macroeconomy to policy interventions.

We believe that our framework opens the door to several intriguing questions which we have left for future research. For instance, what new tools could policy markers use to spur aggregate growth? While demand-side growth policies have been debated in policy circles (see e.g. European Commission, 2003), they have been largely absent from systematic academic analysis within state-of-the-art models of endogenous growth.

Our results also suggest that economies with different firm life-cycle growth profiles and consumption spending patterns may have very different aggregate growth dynamics. What does this imply for the efficacy of pro-growth policies in developing economies, in which markets are potentially more segmented and customer accumulation more frictional compared to developed countries? Or can changes in income inequality affect long-run growth because of associated movements in consumption expenditure allocations?

Finally, our framework studies cost-saving (process) innovations. In the presence of customer accumulation, it would be particularly interesting to also study "product" innovation and understand how these two types of technological progress interact with each other and with firms' life-cycle incentives to attract customers.

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